



FINAL MARK

GIRRAWEEN HIGH SCHOOL

MATHEMATICS

TRIAL Higher School Examination 2013

ANSWERS COVER SHEET

Name: _____

QUESTION	MARK	H2	H3	H4	H5	H6	H7	H8	H9
Section 1	/10								v
	/10								
11	/15		v		v				v
12	/15				v				v
13	/15		v		v				v
14	/15	v	v		v				v
15	/15	v	v	v		v	v	v	v
16	/15			v	v	v	v	v	v
TOTAL	/100	/30	/60	/30	/75	/30	/30	/30	/100

HSC Outcomes

Mathematics

- H1 seeks to apply mathematical techniques to problems in a wide range of practical contexts.
- H2 constructs arguments to prove and justify results.
- H3 manipulates algebraic expressions involving logarithmic and exponential functions.
- H4 expresses practical problems in mathematical terms based on simple given models.
- H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.
- H6 uses the derivative to determine the features of the graph of a function.
- H7 uses the features of a graph to deduce information about the derivative.
- H8 uses techniques of integration to calculate areas and volumes.
- H9 communicates using mathematical language, notation, diagrams and graphs.



2013
TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION
GIRRAWEEN HIGH SCHOOL

MATHEMATICS

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided for this
- Show all necessary working in Questions 11-16

Total marks - 100

Section 1 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes

section

Section 2 90 marks

- Attempt Questions 11 - 16
- Allow about 2 hour 45 minutes for this section

Section 1 Circle the letter corresponding to the correct answer.

(1) What is 4.09784 correct to three significant figures?

- A) 4.09 B) 4.10 C) 4.097 D) 4.098

2) The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β .
What is the value of $\alpha\beta + (\alpha + \beta)$?

- A) 4 B) 2 C) -4 D) -2

3) What is the perpendicular distance of the point (2,-1) from the line

$y = 3x + 1$?

- A) $\frac{6}{\sqrt{10}}$ B) $\frac{6}{\sqrt{5}}$ C) $\frac{8}{\sqrt{10}}$ D) $\frac{8}{\sqrt{5}}$

4) Let $a = e^x$

Which expression is equal to $\log_e(a^2)$?

- A) e^{2x} B) e^{x^2} C) $2x$ D) x^2

5) What is the value of $\int_2^6 \frac{1}{x+2} dx$?

- A) $\ln 2$ B) $\ln 4$ C) $\ln 6$ D) $\ln 8$

6) What values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

- 7) The table below shows the values of a function $f(x) = \sqrt{25 - x^2}$ for six values of x .

x	0	1	2	3	4	5
$f(x)$	5.00	4.90	4.58	4.00	3.00	0.00

What value is an estimate for $\int_0^5 \sqrt{25 - x^2} dx$ using trapezoidal rule with these six values?

- (A) 10.74
(B) 12.65
(C) 18.98
(D) 37.96
- 8) The area under the curve $y = \frac{5}{\sqrt{x}}$, for $1 \leq x \leq e^3$, is rotated about the x axis. What is the exact volume of the solid of revolution?
- (A) 15π units³
(B) 25π units³
(C) 28π units³
(D) 75π units³
- 9) What is the solution to the equation $\log_e(x+2) - \log_e x = \log_e 4$?

- (A) $\frac{2}{5}$
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) $\frac{5}{2}$

10) What is the value of $\sum_{r=1}^{40} (3r - 7)$?

- (A) 109
(B) 2180
(C) 2260
(D) 2380

Section 11 (90 marks) Write your answers on the paper provided.

QUESTION 11(15 marks)

(marks)

- (a) Express 63° in radians correct to 3 significant figures. 2
- (b) Given $(3 + \sqrt{2})^2 = a + \sqrt{b}$, find a and b . 2
- (c) Solve $|3x - 1| < 2$ 2
- (d) Find the vertex and focus of $(x + 2)^2 = -12y + 6$ 2
- (e) Differentiate $y = x^2 \cos 4x$ 3
- (f) The area of a sector of a circle of radius 6cm is 50cm^2 .
Find the length of the arc of the sector. 2
- (g) Find $\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2} dx$ 2

Question 12(15 marks)

(a) Differentiate with respect to x

(i) $y = \ln\left(\frac{x-1}{x^2}\right)$ 2

(ii) $y = e^{4x} + \frac{1}{x}$ 2

(b) Find $\int \frac{4x}{x^2 + 6} dx$ 2

(c) Maureen is raising money for charity by jumping on a Pogo Stick.

Her challenge is to jump between two points, A and B, 20 times. On her first attempt she takes 45 jumps. On her second attempt she takes 48 jumps. On her third attempt she takes 51 jumps. She continues this pattern for all 20 attempts.

(i) How many jumps did she make on her 20th attempt? 2

(ii) How many jumps did she make altogether? 2

Question 12 continued

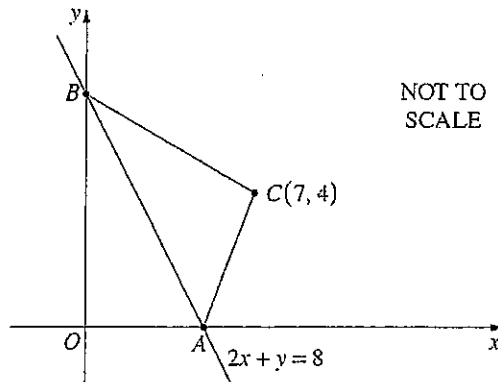
(d) Solve for x $2 \log(x-1) + \log x - \log 4x = 0$ 3

(e) Find the non-zero values of x for which the geometric series

$2 - 6x + 18x^2 - \dots$ has a limiting sum. 2

Question 13(15 marks)

(a) The diagram shows a triangle ABC . The line $2x + y = 8$ meets the x and y axes at the points A and B respectively. The point C has coordinates $(7,4)$.



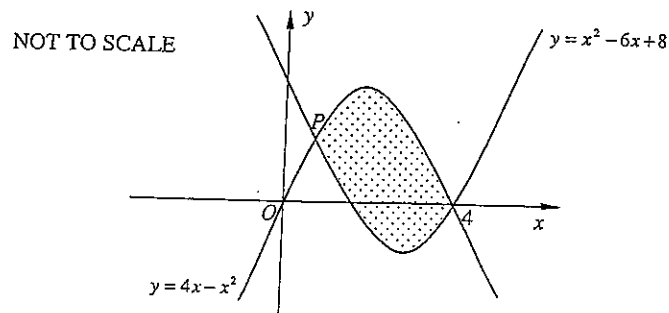
(i) Calculate the distance AB . 2

(ii) It is known that $AC = 5$ and $BC = \sqrt{65}$ (Do not prove this.)

Calculate the size of $\angle ABC$ to the nearest degree. 2

(iii) The point N lies on AB such that CN is perpendicular to AB . Find the coordinates of N . 3

(b) The graphs of the functions $y = 4x - x^2$ and $y = x^2 - 6x + 8$ are shown in the diagram below. They intersect at P and $(4,0)$.



(i) Show that the coordinates of P is $(1,3)$. 1

(ii) Calculate the area of the shaded region. 3

Question 13 continued

(c) (i) Sketch the graph of $y = 3 \sin 2x$ for $0 \leq x \leq 2\pi$ (Your diagram should take up at least one quarter of the page) 2

(ii) Draw the line $y = \frac{x}{2}$ on the same diagram. 1

(iii) Hence find the total number of solutions there are to the equation

$$6 \sin 2x - x = 0 \quad \text{for all } x \geq 0 \quad \text{1}$$

(NB You are **not** required to solve the equation.)

Question 14 (15 marks)

(a) The quadratic equation $x^2 - 3x - 13 = 0$ has roots α and β .

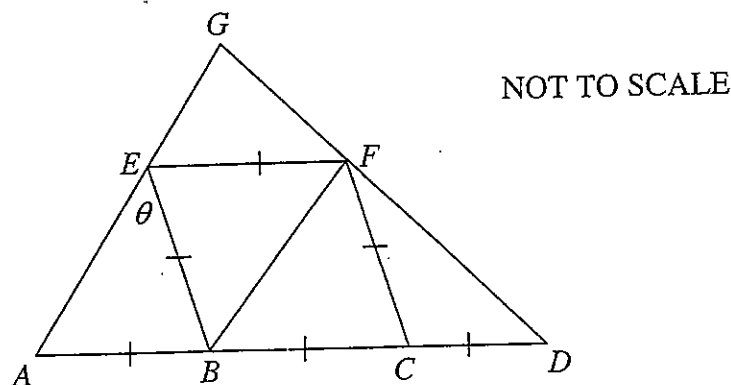
(i) Write down the values of $(\alpha + \beta)$ and $\alpha\beta$. 1

(ii) What is the exact value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ 2

(iii) What is the value of $(6\beta - 2\beta^2)$? 1

(b) $ABCD$ is a straight line such that $AB = BC = CD$.

$EFBC$ is a rhombus and $\angle AEB = \theta$



(i) Show that $\angle EBC = 2\theta$ 1

(ii) The diagonal BF is drawn in the rhombus $EFBC$. Show that

BF is parallel to AE . 2

(iii) Hence or otherwise, find the size of $\angle AGD$. 2

Question 14 continued

(c) Vincent travels to school every day by train. He has found that the probability he misses his train on Mondays is 30% while on Tuesdays it is 20%.

(i) Draw a probability tree diagram for Monday and Tuesday. 2

(ii) Find the probability that Vincent misses the train on ONE of these days. 2

(d) If $a = \log_{\frac{1}{x}} N$ such that $x > 0$ and $N > 0$, show that $a = \log_x N$ 2

Question 15 (15 marks)

(a) Consider the curve $y = x^3 + 6x^2 - 36x + 15$

(i) Find the stationary points and determine their nature 3

(ii) Find the point of inflexion. 1

(iii) Sketch the curve for $-4 \leq x \leq 4$ 2

(iv) For what values is the curve concave down? 1

(b) Use Simpson's Rule with 5 function values to find an

approximate value of $\int_2^4 \frac{x}{2} \ln x dx$ 3

(c) A ship sails from a Port A 50 nautical miles due east to a Port B . It then proceeds a distance of 30 nautical miles on a bearing of $020^\circ T$ to Port C .

(i) Find the distance of Port C from Port A (correct to 2 decimal places). 2

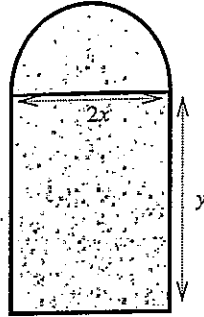
(ii) Find the bearing of Port C from Port A . (to the nearest degree) 2

(d) Find the exact value of $\tan \frac{2\pi}{3}$ 1

Question 16 (15 marks)

- (a) A stained glass window in the local chapel is being constructed.
Its steel frame is in the shape of a rectangle and semi-circle as shown in the diagram.

Let the dimensions of the rectangle be $2x$ metres and y metres.
The length of steel for the frame is to be 4π metres.



NOT TO SCALE

(i) Show that $y = 2\pi - 2x - \frac{\pi x}{2}$. 2

- (ii) Hence show that the area, A , of the window is given by

$$A = 4\pi x - \left(4 + \frac{\pi}{2}\right)x^2 \text{ square metres.} \quad 2$$

- (iii) Hence, find the radius of the semi-circle which maximises the area of the stained glass window. 2

(b) Solve $2\sin^2 \alpha - \sin \alpha = 0$ for $0 < \alpha < 2\pi$. 2

- (c) (i) By completing the square, or otherwise, show that the vertex of the parabola $x^2 - 10x + 15 = 2y$ is at $(5, -5)$. 1

- (ii) Write down the coordinates of focus of this parabola 2

- (d) A particle starts to move from the origin with a velocity $2m/s$ along the x axis.

Its acceleration at any time t seconds is given by : $a = \frac{6}{3t+4}$.

- (i) Show that the velocity, v , of the particle at time t is: 2

$$v = 2 \ln \left(\frac{3}{4}t + 1 \right) + 2m/s \quad 2$$

- (ii) Find the time taken by the particle to reach a velocity $4m/s$. 2