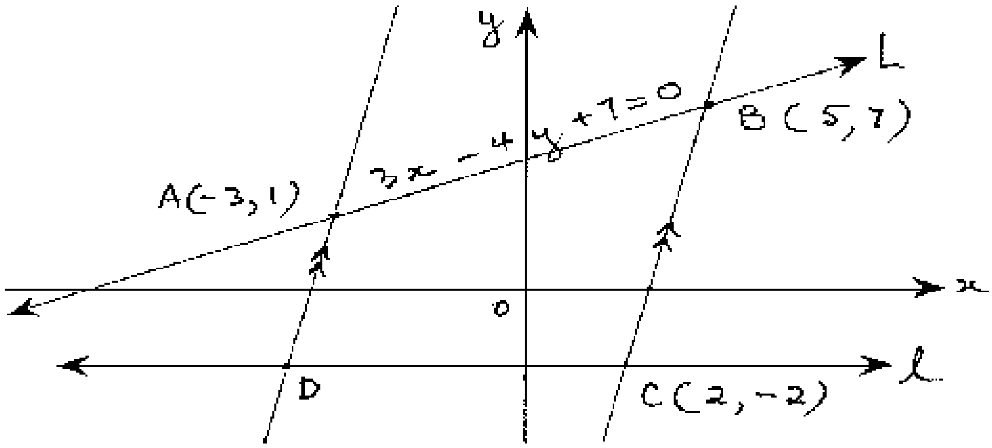
| **Question 1** | | **Marks** |
|---|---|---|
| (a) | Evaluate $\sqrt{\frac{40}{3}-\sqrt{12}}$, correct to three significant figures. | 1 |
| (b) | Find the exact value of $\sin\frac{4p}{3}$. | 1 |
| (c) | Differentiate $\frac{1}{e^x}+\sqrt{x}$ with respect to *x*. | 2 |
| (d) | Solve for *x*, $5=\frac{6x}{x+1}$ | 2 |
| (e) | Find the primitive of $3\sin x$ | 2 |
| (f) | Solve the inequality $\lvert x-1\rvert>3$. | 2 |
| (g) | Given $\log_a 3=1.6$ and $\log_a 7=2.4$, find $\log_a(21a)$ | 2 |

| **Question 2** | | |
|---|---|---|
| (a) | Find the equation of the normal on the curve $y=\ln(x+2)$ at the point (0,ln2) | 3 |
| (b) | Differentiate the following: | |
| | (i) $x^2\tan 5x$ | 2 |
| | (ii) $\frac{x}{1-3x}$ | 2 |
| | (iii) $\sin^3 x$ | 1 |
| (c) | The angle subtended at the centre, *O*, of a sector is 42° and whose radius is 10 cm. find the arc length to the nearest centimetre. | 2 |
| (d) | State the domain and range of the function $f(x)=2\sqrt{x-1}+3$ | 2 |

**Question 3** **Marks**

(a)




The points $A(-3,1)$ and $B(5,7)$ lie on the line L with the equation $3x - 4y + 7 = 0$.
The line $l$ is parallel to the $x$-axis.
The points $C(2,-2)$ and $D$ are two points on $l$ such that $DA \parallel CB$

(i) Find the distance $AB$. 1

(ii) Find the perpendicular distance of $C$ to the line $L$. 2

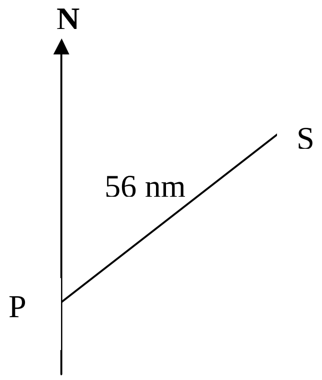
(iii) Find the angle of inclination that line $L$ makes with the $x$-axis (to nearest degree). 2

(iv) Show that the equation of the line passing through $A$ and $D$ is $y = 3x + 10$. 2

(v) Find the coordinates of point $D$. 1

(vi) Find the area of the quadrilateral $ABCD$ by joining $AC$. 2

(b)




A ship $S$ sails from port $P$ on a bearing of N60°E for 56 nautical miles, as shown in the diagram, while a boat $B$ leaves port $P$ on a bearing of 110°T for 48 nautical miles. Calculate the distance from $S$ to $B$ (correct to one decimal place) 2

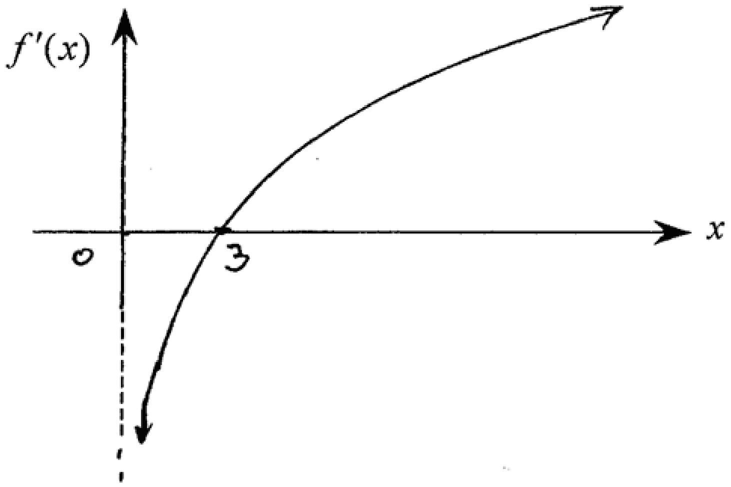
**Question 4** **Marks**

(a) (i) Find $\int \frac{3x^3 - 1}{x} dx$. 2

(ii) Evaluate $\int_0^{\frac{1}{2}} \cos(px) dx$. 2

(b) Solve $\cos 2x = \frac{1}{\sqrt{2}}$ for $0 \le x \le p$ . 2

(c) The sketch of the curve $y = f'(x)$ is given below. 3




Sketch the curve $y = f(x)$, given $f(3) = 0$

(d) The rate of water flowing, $R$ litres per hour, into a pond is given by

$$R = 65 + 4t^{\frac{1}{3}}$$

(i) Calculate the initial flow rate 1

(ii) Find the volume of water in the pond when 8 hours have elapsed, if initially there was 15 litres in the pond. 2

**Question 5** **Marks**

(a) The roots of the equation $x + \frac{1}{x} = 5$ are $a$ and $b$.

Find the value of

(i) $a + \frac{1}{a}$ 1

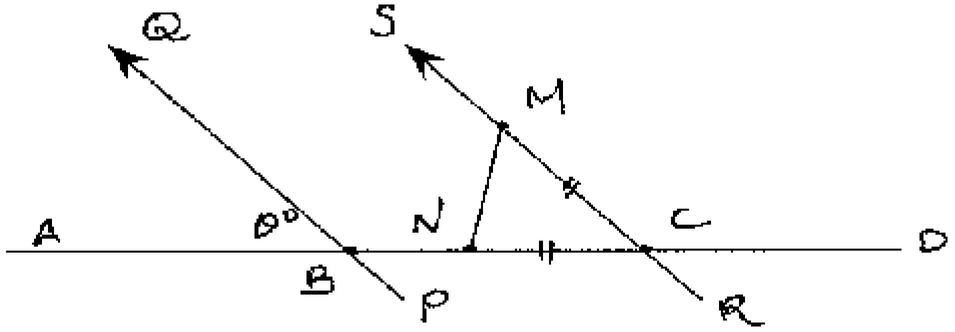
(ii) $a + b$ 2

(iii) $\alpha^2 + \beta^2$ 2

(b) (i) Find the discriminant of $3x^2 + 2x + k$ 1

(ii) For what values of k does the equation $3x^2 + 2x + k = 0$, have real roots? 2

(c) 3




Given $PQ \parallel RS$, $CN$=$CM$ and $\angle ABQ$=$q°$.

Find angle $NMS$ in terms of $q°$, giving reasons.

(d) Given the equation of a parabola is $(x-3)^2 = 4y + 8$,

(i) Find the coordinates of the vertex. 1

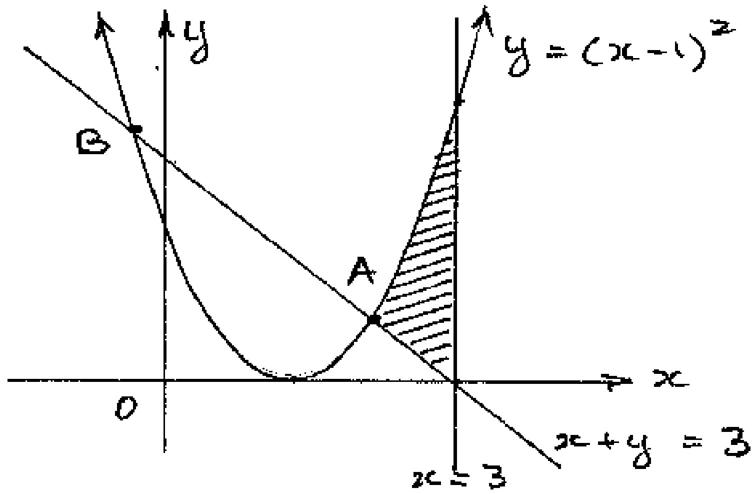
(ii) Find the coordinates of its directrix 1

**Question 6** **Marks**

(a) (i) Solve the equation $x^2 - 3x - 18 = 0$ 2

(ii) Hence, or otherwise find all real solutions to $(x^2+1)^2 - 3(x^2+1) - 18 = 0$ 2

(b) Given the curves $y = (x-1)^2$ and $x + y = 3$ intersect at $A$ and $B$.




(i) Verify that coordinates of $A$=(2,1) 1

(ii) Hence find the area enclosed by the curve $y = (x-1)^2$, and the lines $x + y = 3$ and $x = 3$ 2

(c) Given $\dfrac{dy}{dx} = e^{1-x}$ and when $x = 1$, $y = 3$, find $y$ as a function of $x$ 2

(d) A metal ball is fired into a tank filled with a thick viscous fluid.
The rate of decrease of velocity is proportional to its velocity $v$ cm s$^{-1}$

Thus $\dfrac{dv}{dt} = -kv$, where $k$=0.07 and $t$ is time in seconds.

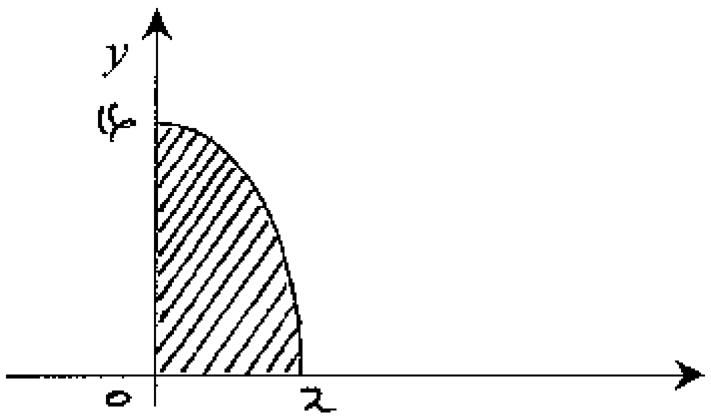
The initial velocity of the ball when it enters the liquid id 85 cm s$^{-1}$

(i) Show that $v = 85e^{-0.07t}$ satisfies the equation $\dfrac{dv}{dt} = -kv$ 1

(ii) Calculate the rate when $t$=5 2

**Question 7** **Marks**

(a) Consider the shaded area of that part of the sketch of the curve $y = 16 - x^4$, for $0 \le x \le 2$, as shown. 3




This area is rotated about the $y$-axis.
Calculate the exact volume of the solid of revolution.

(b) In a game of chess between two players $X$ and $Y$, both of approximately equal ability, the player with the white pieces, having the first move, has a probability of 0.5 of winning, and the probability that the player with the black pieces, for that game, winning is 0.3

(i) What is the probability that the game ends in a draw? 1

(ii) The two players $X$ and $Y$ play each other in a chess competition, each player having the white pieces once.
In the competition the player who wins a game scores 3 points, a player who loses a game scores 1 point and in draw each player receives 2 points.
By drawing a probability tree diagram or otherwise, find the probability that as a result of these two games

($\alpha$) $X$ scores 6 points 1

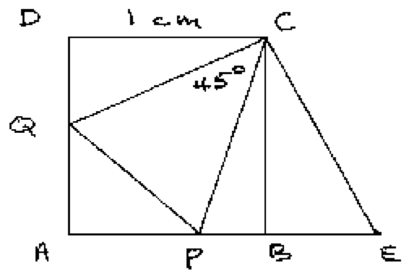
($\beta$) $X$ scores less than 4 points 2

(c) (i) State a formula for the interior angle sim of an $n$-sided convex polygon. 1

(ii) The interior angles of a convex polygon are in arithmetic sequence. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon. 4

**Question 8** **Marks**

(a) In the diagram, *ABCD* is a square of side length 1 cm.




*Not to scale*

Points $P$ and $Q$ lie on $AB$ and $AD$ respectively, and $\angle PCQ = 45°$.
$AB$ is produced to $E$ such that $BE = DQ$ as shown.

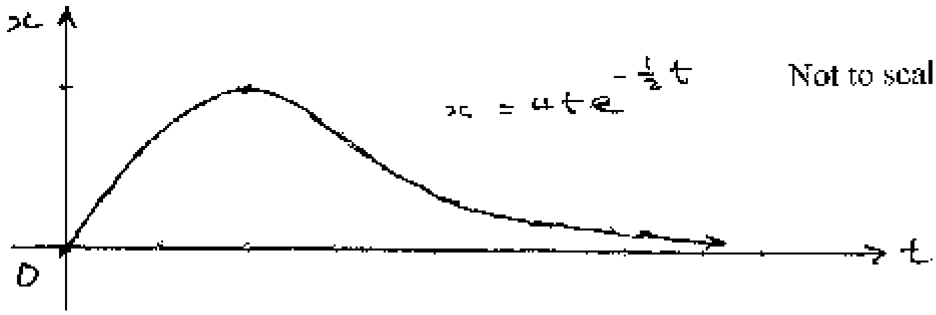
(i) State which test confirms $\Delta CBE \equiv \Delta CDQ$ 1

(ii) Prove that $PC$ bisects $\angle QCE$, giving reasons 2

(iii) Deduce that $PC \perp QE$ (justify) 2

(b) A particle is moving in straight-line motion. The particle starts from the origin and after a time of $t$ seconds it has a displacement of $x$ metres from $O$ given by

$x = 4te^{-\frac{1}{2}t}$ as shown in the diagram.




Its velocity, $v$ m/s, is given by $v = 2(2-t)e^{-\frac{1}{2}t}$

(i) What is the initial velocity? 1

(ii) When and where will the particle be at rest? 2

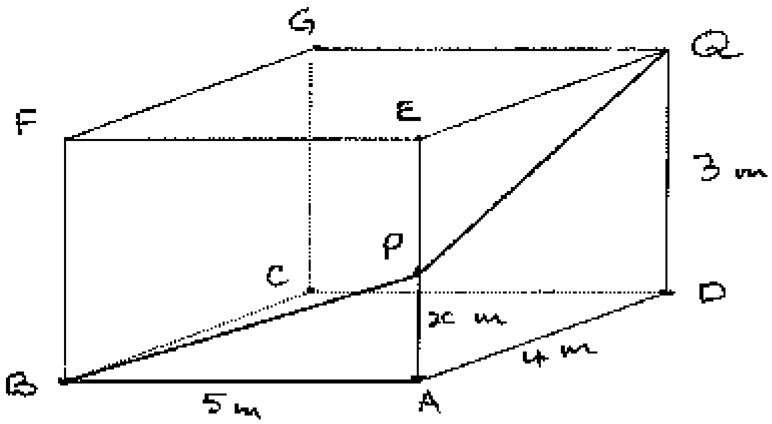
(iii) At what time will the particle be travelling at constant velocity? Give reasons. 3

(iv) When will the particle be accelerating? 1

**Question 9** **Marks**

(a) Show that $\dfrac{d}{dq}\left[\dfrac{1}{\cos q}\right] = \sec q \tan q$ . 2

(b) Fibre cabling is to be laid in a rectangular room along *BP* and *PQ* from the corner *B* of the floor *ABCD* as shown in the diagram.




Given the dimensions of the room are *AB* = 5 m, *AD* = 4 m and the height of the room *AE* = 3 m.

Suppose *AP* = $x$ m,

(i) State the length of *BP* in terms of $x$. 1

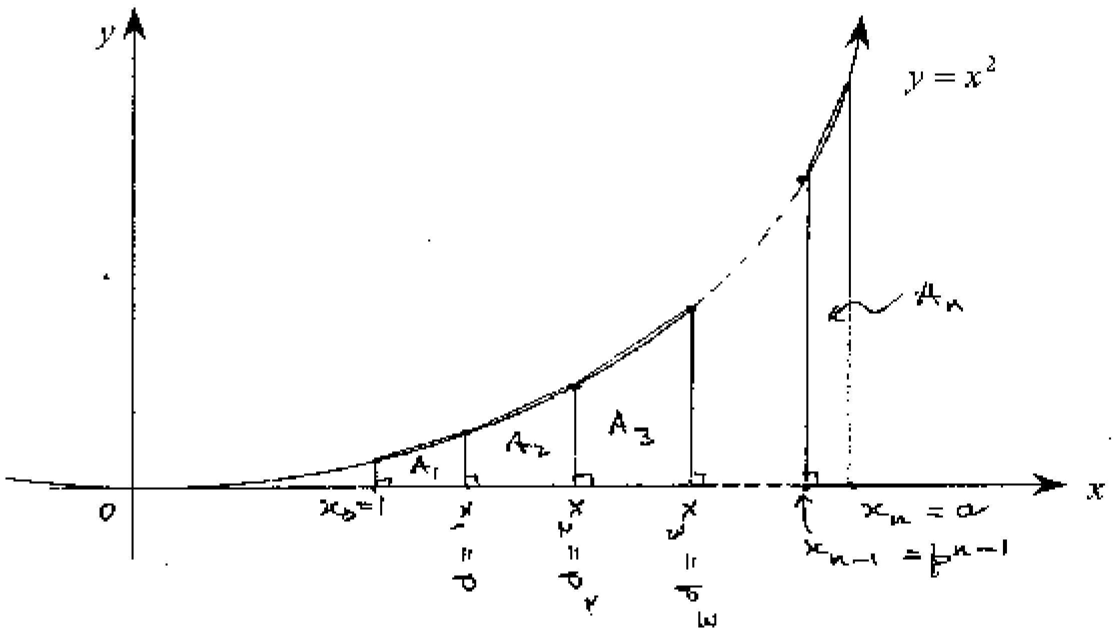
(ii) Show that the length of *PQ* is $\sqrt{25-6x+x^2}$ m. 1

(iii) Hence state the total length, *L* m, of the cabling (in terms of $x$) 1

(iv) Find the value of *AP* when the total length *L* is to be minimum 7

**Question 10** **Marks**

Consider the curve $y = x^2$ for $x \geq 0$, and let $I = \int_1^a x^2 dx$, where $a > 1$.




Divide the interval $1 \leq x \leq a$ into $n$ parts where the divisions are not of equal length, so that $x_0 = 1$, $x_1 = p$, $x_2 = p^2$, …, $x_k = p^k$ and $x_n = a$, where $p^n = a$ and where p > 1.

Let $A_n$ be the area of the $n^{th}$ trapezium, as shown in the diagram.
Let $S_n$ be the sum of the areas of the first $n$ trapezia.

(a) Using the trapezoidal rule, find $S_1$, the area of the first trapezium (in terms of $p$). 2

(b) Given $A_1 = S_1$, show that

(i) $S_2 = S_1 + \frac{1}{2} p^3 (p-1)(1+p^2)$ and *hence* 2

(ii) $S_3 = \frac{1}{2}(p-1)(1+p^2)(1+p^3+p^6)$ 2

(c) Find an expression for $S_n$ and *hence* show that 3

$S_n = \frac{1}{2}(1+p^2)\left(\frac{p^{3n}-1}{p^2+p+1}\right)$, when simplified.

(d) Show that $p \to 1$ as $n \to \infty$. 1

Hence, evaluate $I$, using $I = \lim_{p \to 1} S_n$ 2