Question 1

(a) 3.14

(b) $-\frac{\sqrt{3}}{2}$

(c) $-e^{-x} + \frac{1}{2\sqrt{x}}$

(d) $x = 5$

(e) $-3\cos x + C$

(f) $x < -2$ or $x > 4$

(g) $\log_a 21a = \log_a 3 + \log_a 7 + \log_a a$

$\log_a 21a = 1.6 + 2.4 + 1$

$= 5$

Question 2

(a) $y = \ln(x+2)$

$\frac{dy}{dx} = \frac{1}{x+2}$

when $x = 0$, $\frac{dy}{dx} = \frac{1}{2}$

$\therefore m_{normal} = -2$

let the equation of the normal be $y - y_1 = m(x - x_1)$

where $x_1 = 0$, $y_1 = \ln 2$, $m = -2$

$\therefore 2x + y - \ln 2 = 0$

(b) (i) $5x^2 \sec^2 5x + 2x \tan 5x$

(ii) $\frac{1}{(1-3x)^2}$

(iii) $3\sin^2 x \cos x$

(c) $l = rq$

$= 10(\frac{42p}{180})$

$= 7.3cm$

(d) $\{x : x \geq 1\}$

$\{y : y \geq 3\}$

Question 3

(a)

(i) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{6^2 + 8^2}$

$= 10 units$

(ii) $d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \frac{6 + 8 + 7}{5}$

$= \frac{21}{5} units$

(iii) $m_2 = -\frac{a}{b} = \frac{3}{4}$

$m_{x-axis} = 0$

$\tan q = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$= \frac{3}{4}$

$\therefore q \approx 37^0$

(iv) $m_{BC} = \frac{7 - -2}{5 - 2} = 3$

$m_{AD} = m_{BC}$

$\therefore m_{AD} = 3$

let the equation of AD be $y - y_1 = m(x - x_1)$

where $x_1 = -3$, $y_1 = 1$ and $m = 3$

$\therefore y - 1 = 3(x + 3)$

$\therefore y = 3x + 10$

(v) now D lies on $y = 3x + 10$ and $y = -2$

$\therefore D(-4, -2)$

(vi)

(b) $SB^2 = PS^2 + PB^2 - 2(PS)(PB)\cos \angle SPB$

$SB^2 = 56^2 + 48^2 - 2(56)(48)\cos 50°$

$\therefore SB = 44.54$ nautical miles

Question 4

(a)

(i) $\int \frac{3x^3 - 1}{x} dx = \int (3x^2 - \frac{1}{x}) dx$

$= x^3 - \ln x + C$

(ii) $\int_0^{\frac{1}{2}} \cos(px) dx = \left[\frac{1}{p} \sin(px) \right]_0^{\frac{1}{2}}$

$= \frac{1}{p}$

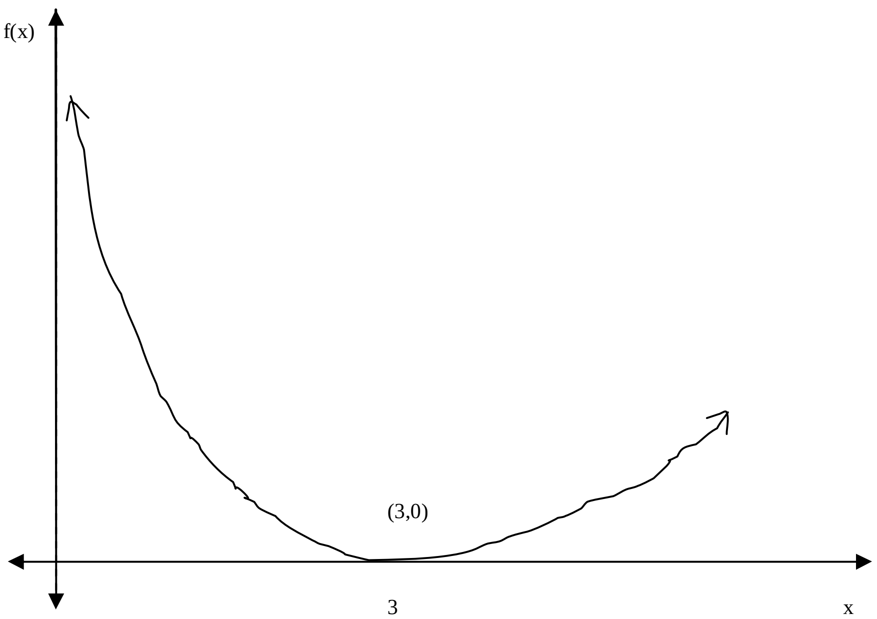
(b)

$\cos 2x = \frac{1}{\sqrt{2}}$

$\therefore 2x = \frac{p}{4} \, or \, \frac{7p}{4}$

$\therefore x = \frac{p}{8} \, or \, \frac{7p}{8}$

(c)



(d)

(i) $R = 65 + 4t^{\frac{1}{3}}$

when $t = 0$, $R = 65 + 4(0)^{\frac{1}{3}} = 65$

(ii) now $R = \frac{dv}{dt} = 65 + 4t^{\frac{1}{3}}$

$\therefore V = 65t + 3t^{\frac{4}{3}} + C$

when t = 0, V = 15, $\therefore C = 15$

$\therefore V = 65t + 3t^{\frac{4}{3}} + 15$

when t = 0, V = 583 litres

Question 5

(a)

(i) $a+\frac{1}{a}=5$

(ii) $a+b=-\frac{b}{a}=5$

(iii) $a^2+b^2=(a+b)^2-2ab$

$=(5)^2-2(1)$

$=23$

(b)

(i) $\Delta=b^2-4ac=4-4(3)(k)$

$=4-12k$

(ii) for real roots $\Delta\geq 0$

$\therefore 4-12k\geq 0$

$\therefore k\leq\frac{1}{3}$

(c)

$\angle ABQ=\angle ACS=q°$ (corresponding angles on $PQ||RS$ are equal)

now $\angle CNM=\angle NMC$ (equal angles opposite equal sides in isosceles triangle PRM)

$\therefore q°+\angle CNM+\angle NMC=180°$ (angle sum triangle CNM is 180°)

$\therefore 2$ x $\angle NMC=180°-q°$ ($\angle NMC=\angle CNM$)

$\therefore \angle NMC=\frac{180°-q°}{2}$

$\angle NMS+\angle NMC=180°$ (adjacent angles on a straight line are supplementary)

$\therefore \angle NMS=180°-\frac{180°-q°}{2}$

$\therefore \angle NMS=\frac{180°+q°}{2}$

(d)

(i) vertex: (h,k)

(3,-2)

(ii) directrix: $y=-a+k$

$\therefore y=-3$

Question 6

(a) (i) $x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$\therefore x = -3$ or $x = 6$

(ii) $(x^2+1)^2 - 3(x^2+1) - 18 = 0$

let $U = x^2 + 1$

$\therefore U^2 - 3U - 18 = 0$

$(U-6)(U+3) = 0$

$\therefore U = -3$ or $U = 6$

$\therefore x^2 + 1 = -3$

$x^2 = -4$

no real solution

$\therefore x^2 + 1 = 6$

$x^2 = 5$

$x = \pm\sqrt{5}$

$\therefore x = \pm\sqrt{5}$

(b) (i) $y = (x-1)^2$ _(1)

$x + y = 3$___(2)

$\therefore x = -1 or 2$

$\therefore y = 1 or 4$

the curves intersect at (2,1) and (-1,4)

(ii) Area = $\int_2^3 [(x-1)^2 - (3-x)]dx$

$= \int_2^3 (x^2 - x - 2)dx$

$= \frac{11}{6}$ units^2

(c) $\frac{dy}{dx} = e^{1-x}$

$y = \int e^{1-x} dx$

$\therefore y = -e^{1-x} + C$

when x=1, y=3

$\therefore 3 = -1 + C$

$\therefore C = 4$

$\therefore y = -e^{1-x} + 4$

(d) (i) $V = 85e^{-0.07t}$

$\frac{dV}{dt} = 85 \times -0.07 \times e^{-0.07t}$

$= -0.07 \times 85e^{-0.07t}$

$= -kV$

(ii) when t = 5, $\frac{dV}{dt} = -0.07 \times 85e^{-0.07\times 5}$

$= -4.19$ cm/s^2

Question 7

(a)

$$V = p\int_a^b x^2\,dy$$

$$V = p\int_0^{16} (16-y)^{\frac{1}{2}}\,dy$$

$$V = -p\int_0^{16} -(16-y)^{\frac{1}{2}}\,dy$$

$$V = -p\left[\frac{2(16-y)^{\frac{3}{2}}}{3}\right]_0^{16}$$

$$V = \frac{128p}{3}\text{ units}^3$$

(b) -

(c) (i) $(n-2)\times 180^\circ$

(ii) $S_n = \frac{n}{2}(2a+(n-1)d)$

$$= \frac{n}{2}(240^\circ + 5^\circ(n-1))$$

$$= \frac{n}{2}(235^\circ) + \frac{5^\circ n^2}{2} = (n-2)\times 180^\circ$$

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$\therefore n = 6 or 19$

Question 8

(a) (i) SAS

(ii) $\angle DCQ + \angle QCP + \angle PCB = 90°$ (interior angle of a square is a right angle)

$\therefore \angle DCQ + \angle PCB = 45°$

now $\angle DCQ = \angle BCE$ (corresponding angles in $\Delta CBE \equiv \Delta CDQ$)

$\therefore \angle BCE + \angle PCB = 45°$

$\therefore \angle QCP = \angle PCE = 45°$

$\therefore$ PC bisects $\angle QCE$

(iii)-

(b) (i) when $t = 0$, $v = 2(2-0)e^0$

$= 4m/s$

(ii) particle is at rest when v = 0

$\therefore 2(2-t)e^{-\frac{t}{2}} = 0$

$\therefore t = 0$

when t = 2, $x = 4(2)e^{-1}$

$= \frac{8}{e}m$

the particle will be at rest when t = 2, and at $x = \frac{8}{e}m$

(iii)–

(iv) particle accelerates when $\frac{d^2x}{dt^2} > 0$

ie when t > 4

Question 9

(a) $\frac{d}{dq}(\frac{1}{\cos q}) = \frac{(\cos q)(0)-(1)(-\sin q)}{\cos^2 q}$

$= \sec q \tan q$

(b) (i) $BP^2 = AB^2 + AP^2$ (by Pythagoras)

$BP^2 = 5^2 + x^2$

$\therefore BP = \sqrt{25+x^2}$ (BP > 0)

(ii) $AE = AP + PE$

$PE = AE - AP$

$PE = 3 - x$

now $PQ^2 = PE^2 + EQ^2$ (by Pythagoras)

$= (3-x)^2 + 4^2$

$= 25 - 6x + x^2$

$\therefore PQ = \sqrt{25-6x+x^2}$ (PQ > 0)

(iii)total cabling = BP + PQ

$L = (\sqrt{25+x^2} + \sqrt{25-6x+x^2})$

(iv) $\frac{dL}{dx} = \frac{1}{2}(25+x^2)^{-\frac{1}{2}} \times (2x) + \frac{1}{2}(25-6x+x^2)^{-\frac{1}{2}} \times (2x-6)$

$= \frac{x}{\sqrt{25+x^2}} + \frac{x-3}{\sqrt{25-6x+x^2}} = 0$ (for stationary points)

$\therefore x = \frac{5}{3} or 15$

now $0 \le x \le 3$

$\therefore x = \frac{5}{3}$

<u>Test</u>

x	1	$\frac{5}{3}$	2
$\frac{dL}{dx}$	-0.25	0	0.129
	\	<u>*MIN*</u>	/

Since the function is continuous in the domain $0 \le x \le 3$, $x = \frac{5}{3}$ is a local minimum and there is only one turning point in the domain, $x = \frac{5}{3}$ is also the absolute minimum

$\therefore AP = \frac{5}{3}$ metres

Question 10

(a) $\int_1^p x^2 dx \approx \frac{p-1}{2}(1+p^2)$

$$= \frac{p-1}{2} + \frac{p^2(p-1)}{2}$$

$$= \frac{p-1}{2}(p^2+1)$$

(b)

(i) $S_2 = S_1 + A_2$

$$= S_1 + \frac{p^2-p}{2}(p^2+p^4)$$

$$= S_1 + \frac{p^4+p^6-p^3-p^5}{2}$$

$$= S_1 + \frac{p^3}{2}(p^3-p^2+p-1)$$

$$= S_1 + \frac{1}{2}p^3(p-1)(1+p^2)$$

(ii) $S_3 = S_2 + A_3$

$$= \frac{(p^2+1)(p-1)}{2} + \frac{p^3(p-1)(1+p^2)}{2} + \frac{p^3-p^2}{2}(p^4+p^6)$$

$$= \frac{(p^2+1)(p-1)}{2}\left[1+p^3+p^6\right]$$

(c) $S_n = \frac{1}{2}(p-1)(1+p^2)\left[1+p^3+p^6+\ldots+p^{3(n-1)}\right]$

$$= \frac{1}{2}(p-1)(1+p^2)\times\frac{\left[1\times(p^3)^n-1\right]}{p^3-1}$$

$$= \frac{1}{2}(1+p^2)\left[\frac{p^{3n}-1}{p^2+p+1}\right]$$

(d) –

(e) -