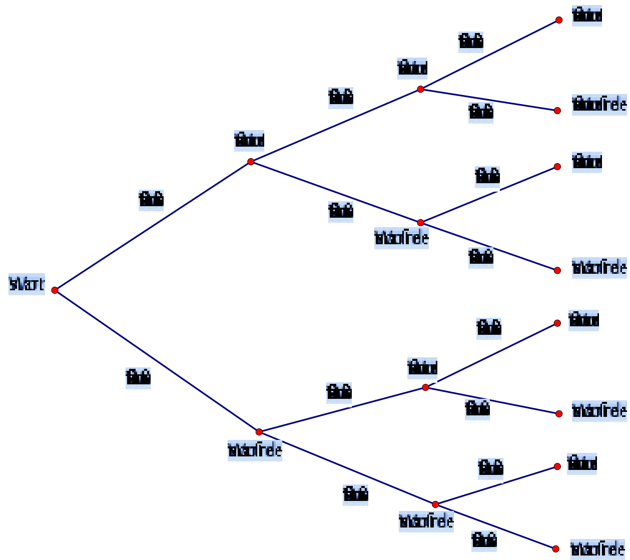
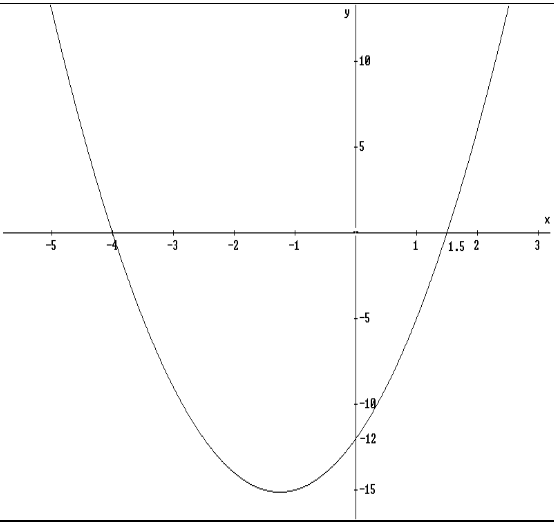
Question 1

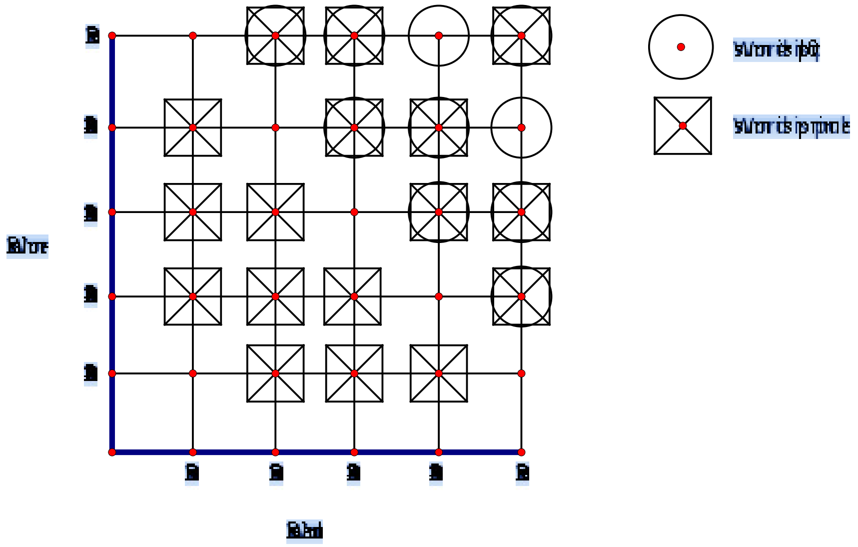
(a)	(i)	when $t = 0$, $2916 = P_0 e^0$ $\therefore P_0 = 2916$ when $t = 10$, $4860 = 2916e^{10k}$ $e^{10k} = \frac{5}{3}$ $10k = \ln\left(\frac{5}{3}\right)$ $k = \frac{1}{10}\ln\left(\frac{5}{3}\right)$	3
	(ii)	when $t = 40$, $P = 2916e^{4\ln\left(\frac{5}{3}\right)}$ $\therefore$ population $= 22\,500$	2
	(iii)	$\frac{dP}{dt} = kP_0e^{kt}$ $= kP$ since $P = P_0e^{kt}$	1
	(iv)	(α) when $P = 600$, $\frac{dP}{dt} = \frac{1}{10}\ln\left(\frac{5}{3}\right)\times 600$ $= 310$ (to 2 sig. fig.)	1
		(β) when $t = 30$, $P = kP_0e^{kt}$ $= \frac{1}{10}\ln\left(\frac{5}{3}\right)\times 2916 \times e^{3\ln\left(\frac{5}{3}\right)}$ $= 690$ (to 2 sig. fig.)	1
(b)	(i)	when $x_A = 0$, $t(30-t)(30+t) = 0$ $t_A = 0$ or ± 30 when $x_B = 0$, $36t(25-t) = 0$ $t_B = 0$ or 25 Time intervals are $0 < t_A < 30$ and $0 < t_B < 25$.	2
	(ii)	$D = x_A - x_B$ $= (900t - t^3) - (900t - 36t^2)$ $D = 36t^2 - t^3$	1

	(iii)	$\frac{dD}{dt} = 72t - 3t^2$ for stat. pt. $\frac{dD}{dt} = 0$ $72t - 3t^2 = 0$ $3t(24 - t) = 0$ $t = 0$ or 24 $\frac{d^2D}{dt^2} = 72 - 6t$ when $t = 0,\ \frac{d^2D}{dt^2} = 72 > 0$ $\therefore$ local min. tp. when $t = 24,\ \frac{d^2D}{dt^2} = -72 < 0$ $\therefore$ local max. tp. when $t = 24,\ D = 36(24)^2 - (24)^3$ $= 6912$ maximum distance is 6912 m.	4
Question 2			
(a)	(i)		1
	(ii)	$P(\text{no free Mars bar}) = (0.8)^3$ or $\left(\frac{4}{5}\right)^3$ $= 0.512$ or $\frac{64}{125}$	1
	(iii)	$P(\text{at laest one free Mars bar}) = 1 - P(\text{no free Mars bars})$ $= 1 - (0.8)^3$ $= 0.488$ or $\frac{61}{125}$	1
	(iv)	$P(\text{one free Mars bar}) = 3 \times (0.2) \times (0.8)^2$ or $3 \times \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$ $= 0.384$ or $\frac{48}{125}$	1

(b)	(i)	if $x = 0$, $y = -12$ if $y = 0$, $2x^2 + 5x - 12 = 0$ $(2x-3)(x+4) = 0$ $x = \frac{3}{2} \text{ or } -4$ 	2
	(ii)	$\{p : p < -4\} \cup \{p : p > 1.5\}$ or $p < -4$ or $p > 1.5$	1
(c)	(i)	Since the expression for velocity is a quadratic with negative leading coefficient, the maximum speed is at the turning point $\therefore\ t = -\frac{10}{-2}$ $= 5$ $v = 24 + 10(5) - (5)^2$ $= 49$ $\therefore$ max speed = 49 *km/hr*	2
	(ii)	at rest when $v = 0$ $-t^2 + 10t + 24 = 0$ $t^2 - 10t - 24 = 0$ $(t-12)(t+2) = 0$ $t = 12 \quad (t \geq 0)$ $a = 10 - 2t$ when $t = 12$, $a = 10 - 24$ $= -14$ $\therefore$ acceleration is $-14\ km/hr^2$	3

	(iii)	$x = 24t + 5t^2 - \frac{1}{3}t^3 + c$ when $t = 0, \quad x = 0$ $0 = 0 + 0 + 0 + c$ $\therefore c = 0$ $x = 24t + 5t^2 - \frac{1}{3}t^3$ when $t = 0, \quad x = 0$ when $t = 12, \quad x = 24(12) + 5(12)^2 - \frac{1}{3}(12)^3$ $= 432$ when $t = 15, \quad x = 24(15) + 5(15)^2 - \frac{1}{3}(15)^3$ $= 360$ distance travelled $= 432 + (432 - 360)\ km$ $= 504\ km$	3
Question 3			
(a)	(i)	At 2 hours	1
	(ii)	At 80m to the right of the origin or at position B	1
	(iii)	$0 \le t < 4$	2
	(iv)	The object is moving to the left with increasing speed.	2
(b)	(i)	when $t = 0, \quad N = 12\ 000$ $12\ 000 = \dfrac{A}{5 + 3e^0}$ $A = 96\ 000$	1
	(ii)	when $t = 8$ $N = \dfrac{96000}{5 + 3e^{-0.1 \times 8}}$ ≈ 15122.9 $= 15100$ (to nearest 100)	1
	(iii)	when $N = 18000$ $18000 = \dfrac{96000}{5 + 3e^{-0.1t}}$ $5 + 3e^{-0.1t} = \frac{16}{3}$ $3e^{-0.1t} = \frac{1}{3}$ $e^{-0.1t} = \frac{1}{9}$ $-0.01t = \ln\left(\frac{1}{9}\right)$ $t = \dfrac{\ln\left(\frac{1}{9}\right)}{-0.1}$ $= 21.97$ time $= 22.0$ years (to 1 d.p.)	2

	(iv)	as $t \to \infty$, $e^{-kt} \to 0$ $\therefore\ N \to \frac{96000}{5}$ $N \to 19\ 200$ population tends to19200 penguins.	1
(c)	(i)	Curves meet when $mx + b = \dfrac{a}{x}$ $mx^2 + bx = a$ $mx^2 + bx - a = 0$ Now for the line to be a tangent, this equation has must have only one solution i.e. the $\Delta = 0$ $\therefore\ \Delta = b^2 - 4m(-a)$ $b^2 - 4m(-a) = 0$ $b^2 + 4ma = 0$	2
	(ii)	Let $m = -3$ and $a = 12$ $\therefore b^2 + 4(12)(-3) \geq 0$ $b^2 - 144 \geq 0$ $(b-12)(b+12) \geq 0$ $b \leq -12$ or $b \geq 12$	2
Question 4			
(a)	(i)	$\alpha + \beta = \frac{5}{3}$	1
	(ii)	$\alpha\beta = \frac{1}{3}$	1
	(iii)	$\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$ $= \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ $= \dfrac{\left(\frac{5}{3}\right)^2 - 2\left(\frac{1}{3}\right)}{\left(\frac{1}{3}\right)}$ $= \frac{19}{3}$	2

(b)	(i)		1
	(ii)	$P(\text{sum} > 10) = \frac{10}{25}$ $= \frac{2}{5}$	1
	(iii)	$P(\text{sum is prime}) = \frac{17}{25}$	1
	(iv)	$P(\text{sum} > 10 \text{ given the sum is prime}) = \frac{8}{17}$	2
(c)	(i)	$\ddot{x} = -(5t+1)^{-2}$ $v = \frac{-(5t+1)^{-1}}{-5} + c_1$ $= \frac{0.2}{5t+1} + c_1$ when $t = 0, v = 1$ $\therefore\ 1 = 0.2 + c_1$ $c_1 = 0.8$ $v = \frac{0.2}{5t+1} + 0.8$	2
	(ii)	as $t \to \infty$ $v \to 0 + 0.8$ limiting speed is 0.8 m/s	1

	(iii)	$x = \frac{0.2\ln(5t+1)}{5} + 0.8t + c_2$ $= 0.04\ln(5t+1) + 0.8t + c_2$ when $t = 0,\ x = -3$ $-3 = 0.04\ln(1) + 0 + c_2$ $c_2 = -3$ $x = 0.04\ln(5t+1) + 0.8t - 3$ when $t = 10$ $x = 0.04\ln(51) + 8 - 3$ ≈ 5.157 position is 5m to the right of the origin.	3