

Year 12 Term 1 Assessment 2005
Mathematics (2 unit)

Question 1: (12 marks)

Marks

(a) Find:

(i) $\int \left(x^2 + \frac{8}{x} \right) dx.$ 2

(ii) $\int \sin 3x dx.$ 2

(iii) $\int \left(e^{6x} + 4x\sqrt{x} \right) dx.$ 2

(b) Evaluate:

(i) $\int_{-2}^4 \frac{6}{3x+8} dx.$ 2

(ii) $\int_0^{\pi} \sec^2\left(\frac{x}{3}\right) dx.$ 2

(c) Find the value of x if $2x-1$, x and $3x+2$ are successive terms in an Arithmetic Progression. 2

Question 2: (12 marks)

(a) On a number plane clearly indicate the region defined by the intersection of the inequalities $y \leq 2 - x$ and $x^2 + y^2 \geq 16$. 4

(b) Find the 100th term in the Arithmetic Progression $\{-29, -25, -21, \dots\}$. 3

(c) (i) Sketch the curve $y = x^3 - 4x^2$ clearly showing any intercepts with the coordinate axes. 2
(You are not required to show the position of the turning points)

(ii) Find the area bounded by the curve $y = x^3 - 4x^2$ and the x -axis. 3

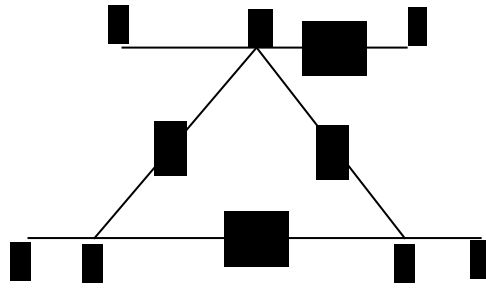
Question 3: (12 marks)

Marks

- (a) Intervals AB and PQ are parallel and $XY = XZ$.

3

Copy the diagram onto your answer sheet and prove that $\angle AXY$ and $\angle XZQ$ are supplementary.



- (b) Does a Geometric Progression exist with a first term equal to 9 and a limiting sum equal to 15? Justify your answer.

3

- (c) (i) Find the points of intersection of the curves $y = 4 - x$ and $y = x^2 - 3x - 4$.

3

- (ii) Find the area bounded by the curves $y = 4 - x$ and $y = x^2 - 3x - 4$.

3

Question 4: (12 marks)

- (a) (i) Sketch the curve $y = 2 - e^x$ clearly showing any intercepts with the coordinate axes.

2

- (ii) Find the volume of the solid formed when the area bounded by the curve $y = 2 - e^x$ and the coordinate axes is rotated one revolution about the x -axis. Write your answer in the form $\pi(A + B \log_e 2)$ where A and B are rational numbers.

3

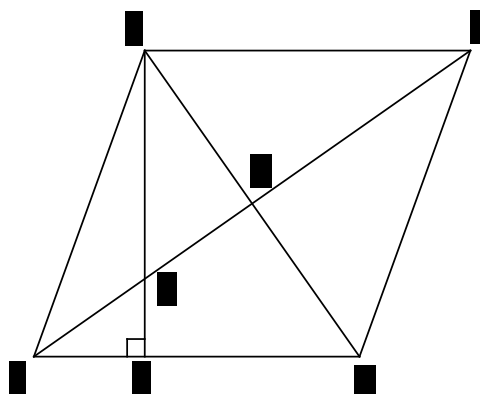
- (b) Find the sum of the first 5 terms of the Geometric Progression $\{32, 24, 18, \dots\}$. Write your answer as a mixed numeral in simplest form.

3

- (c) $ABCD$ is a rhombus whose diagonals intersect at X . AP is perpendicular to BC and intersects diagonal BD at Q .

4

Copy the diagram onto your answer sheet and prove that $\angle PAC = \angle ADB$.



Question 5: (12 marks)

Marks

- (a) (i) Sketch the curve $y = 8 - x^3$ clearly showing any intercepts with the coordinate axes. 2
- (ii) Find the volume of the solid formed when the area bounded by $y = 8 - x^3$ and the positive coordinate axes is rotated one revolution about the y -axis. 3
- (b) (i) Peter Perfect sets up a bank account to save for a holiday. He decides to deposit \$100 in the account at the start of each month commencing on February 1st 2005. Interest at a rate of 3% p.a. is paid at the end of each month on the balance in the account.
- (α) Prove that the value $\$A_n$ of the account at the end of the n^{th} month is given by the formula $A_n = 40100(1.0025^n - 1)$. 2
- (β) How many months will Peter need to contribute to the account if he needs \$3000 for his holiday? Give your answer correct to the nearest month. 2
- (ii) In part (i) Peter decides to increase each monthly deposit by 2% with the first increase in March 2005.
- (α) Find a formula for the value $\$B_n$ of the account at the end of the n^{th} month. 2
- (β) Find the value of the account at the end of 24 months. Give your answer correct to the nearest dollar. 1

◆ THIS IS THE END OF THE EXAMINATION QUESTIONS ◆

The table below shows the first three stages in a pattern constructed using buttons.

2

1

Number of Buttons	1	1	1

- (i) Find the number of buttons needed for the 10th pattern (P_{10}).
- (ii) What is the greatest number of different stages can be illustrated using 1000 buttons?

- (i) The area bounded by the curve $y = \log_3 x$, the coordinate axes and the line $y = 2$ is rotated one revolution about the y -axis. Find the volume of the solid formed. You may assume that $a^p = e^{p \ln a}$

4

- (i) On the same set of axes, sketch the curves $y = 2 \sin x$ and $y = \sin 2x$ for $0 \leq x \leq \pi$.

2

- (ii) Find the area bounded by $y = 2 \sin x$ and $y = \sin 2x$ for $0 \leq x \leq \pi$.

2

- (i) Find the volume of the solid formed when the area bounded by the curve $y = (x - 2)^2$ and the positive coordinate axes is rotated one revolution about the y -axis.

2