**Year 12 Term 1 Assessment 2005 – Mathematics**

**Question 1:**

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| (a) | $(i)\ \int\left(x^2+\frac{8}{x}\right)dx=\frac{1}{3}x^3+8\ln x+c$ | |
| | $(ii)\ \int \sin 3x\ dx=-\frac{1}{3}\cos 3x+c$ | |
| | $(iii)\ \int\left(e^{6x}+4x^{\frac{3}{2}}\right)dx=\frac{1}{6}e^{6x}+\frac{8}{5}x^{\frac{5}{2}}+c$ | |

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|---|---|---|
| (b) | $(i)\ \int_{-2}^{4}\frac{6}{3x+8}\,dx=\left[2\ln(3x+8)\right]_{-2}^{4}$ <br> $=2\ln 20-2\ln 2$ <br> $=2\ln 10$ | |
| | $(ii)\ \int_0^{\pi}\sec^2\left(\frac{x}{3}\right)dx=\left[3\tan\left(\frac{x}{3}\right)\right]_0^{\pi}$ <br> $=3\tan\left(\frac{\pi}{3}\right)-3\tan 0$ <br> $=3\sqrt{3}$ | |

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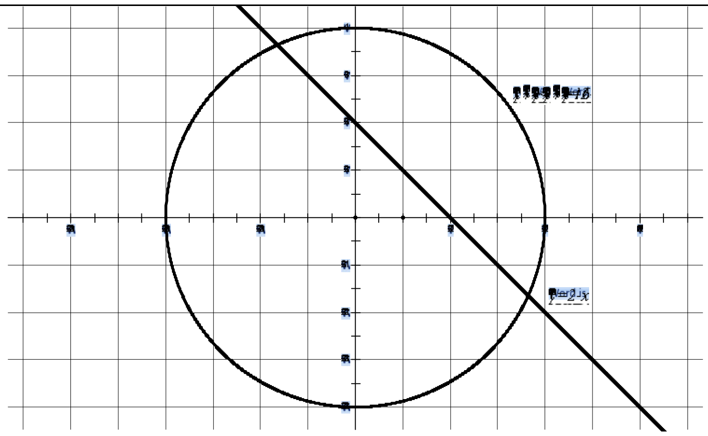
|---|---|---|
| (c) | $(x)-(2x-1)=(3x+2)-(x)$ <br> $-x+1=2x+2$ <br> $3x=-1$ <br> $x=-\frac{1}{3}$ | |

**Question 2:**

(a)

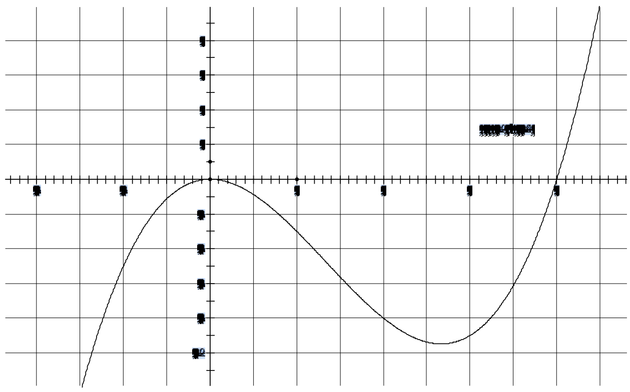


(b)

$a = -29,\ d = 4$

$$T_{100} = -29 + 99(4)$$
$$= 367$$

(c) $(i)$

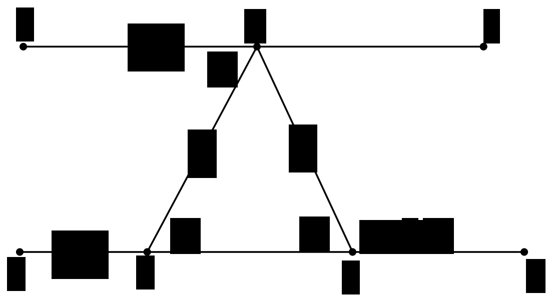


$$(ii)\ A = \left| \int_0^4 \left(x^3 - 4x^2\right) dx \right|$$
$$= \left| \left[ \frac{1}{4}x^4 - \frac{4}{3}x^3 \right]_0^4 \right|$$
$$= \left| \left( \frac{4^4}{4} - \frac{4 \times 4^3}{3} \right) - (0) \right|$$
$$= \frac{64}{3}$$

$$\text{Area} = 21\frac{1}{3}\,u^2$$

**Question 3:**

(a)




$Let\ A\hat{X}Y = \theta°$

$X\hat{Y}Z = \theta°$ (alternate angles are equal as $AB \parallel PQ$)

$X\hat{Z}Y = \theta°$ (equal angles are opposite equal sides in $\Delta XYZ$)

$X\hat{Z}Q + \theta° = 180°$ (straight angle $P\hat{Z}Q = 180°$)

$X\hat{Z}Q = 180° - \theta°$

$\therefore A\hat{X}Y$ and $X\hat{Z}Q$ are supplementary

(b)

$$S = \frac{a}{1-r}$$

$$15 = \frac{9}{1-r}$$

$$15 - 15r = 9$$

$$15r = 6$$

$$r = \frac{2}{5}$$

since $-1 < r < 1$ then sequence exists

(c)

$(i)$ $4 - x = x^2 - 3x - 4$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = -2$ or $4$

$\therefore$ points are $(-2,6)$ and $(4,0)$

$(ii)$ $A = \int_{-2}^{4} \{(4-x) - (x^2 - 3x - 4)\}\, dx$

$= \int_{-2}^{4} (8 + 2x - x^2)\, dx$

$= \left[ 8x + x^2 - \frac{1}{3}x^3 \right]_{-2}^{4}$

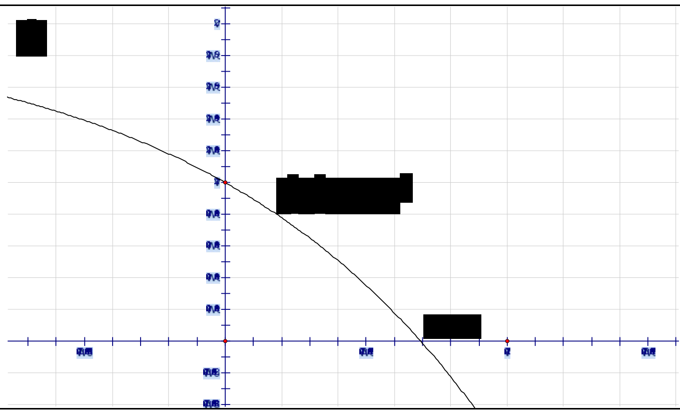
$= 26\frac{2}{3} - \left( -9\frac{1}{3} \right)$

$= 36$

$\therefore$ area $= 36u^2$

**Question 4:**

(a)



$$(ii)\, V=\pi\int y^2\ dx$$

$$=\pi\int_0^{\ln 2}\left(2-e^x\right)^2\ dx$$

$$=\pi\int_0^{\ln 2}\left(4-2e^x+e^{2x}\right)dx$$

$$=\pi\left[4x-2e^x+\frac{1}{2}e^{2x}\right]_0^{\ln 2}$$

$$=\pi\left\{\left(4\ln 2-2e^{\ln 2}+\frac{1}{2}e^{2\ln 2}\right)-\left(0-e^0+\frac{1}{2}e^0\right)\right\}$$

$$=\pi\left\{\left(4\ln 2-2\times 2+\frac{1}{2}\times 4\right)+\frac{1}{2}\right\}$$

$$=\pi\left(-\frac{5}{2}+4\ln 2\right)$$

$$\therefore \text{Volume}=\pi\left(-\frac{5}{2}+4\ln 2\right)u^3$$

(b)

$$S_5=\frac{32\left\{1-\left(\frac{3}{4}\right)^5\right\}}{1-\frac{3}{4}}$$

$$=\frac{128\left(4^5-3^5\right)}{4^5}$$

$$=97\frac{5}{8}$$

(c)

$BC \parallel DC$ (opposite sides of rhombus $ABCD$ are parallel)

Let $\angle ADB=\alpha°$

$\angle DBC=\alpha°$ (alternate angles are equal as $BC \parallel DC$)

$\angle BXC=90°$ (diagonals of rhombus $ABCD$ are perpendicular)

$\angle BCX+\alpha°+90°=180°$ (angle sum of $\Delta BCX=180°$)

$\angle BCX=(90-\alpha)°$

$\angle PAC+90°+(90-\alpha)°=180°$ (angle sum of $\Delta APC=180°$)

$\angle PAC=\alpha°$

$\therefore \angle ADB=\angle PAC$

Question 5:

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