YEAR 12 MATHEMATICS ASSESSMENT TEST TERM 2
2005

1(a) $2x^2+3x-4=0$

(i) $\alpha+\beta=-b/a = -3/2$ ①

(ii) $\alpha\beta = c/a = -2$ ①

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$

$= \frac{9}{4}+4$

$= 6\frac{1}{4}$ ①

1(b) (i) $V = Ae^{-kt}$

$\frac{dV}{dt} = -kAe^{-kt}$

$LHS = \frac{dV}{dt}+kV = -kAe^{-kt}+kAe^{-kt}$ ①

$= 0$

$= RHS$

$\therefore V = Ae^{-kt}$ is a solution

(ii) $t=0 \quad V=A$

$t=1 \quad V=\frac{1}{2}A$

$\therefore \frac{1}{2}A = Ae^{-k\times 1}$

$\therefore \frac{1}{2} = e^{-k}$

$\therefore \ln\frac{1}{2} = -k$ ①

$\therefore k = \ln 2$ /week

(iii) $t=4 \quad V=5000e^{-4k}$

$=5000e^{-4\ln 2}$

$= 312.5$

$\therefore$ Vol lost $= 5000-312.5$

$= 4687.5L$

$= 4688L$ (nearest Litre)

1(c) (i)

(α) $P(J)=\frac{3}{4} \qquad P(S)=\frac{1}{3}$

$P(\text{Both}) = \frac{3}{4}\times\frac{1}{3} = \frac{1}{4}$ ①

(β) $P(\text{at least 1 hit}) = 1-P(\text{Both miss})$

$= 1-\frac{1}{4}\times\frac{2}{3}$

$= 1-\frac{1}{6} = 5/6$ ①

(ii) $P(S\text{ wins}) = P(\underset{\text{miss}}{S})\times P(\underset{\text{miss}}{J})\times P(\underset{\text{HITS}}{S})$

$= \frac{2}{3}\times\frac{1}{4}\times\frac{1}{3}$ $\left(1\frac{1}{2}\right)$

$= \frac{1}{18}$ $\left(\frac{1}{2}\right)$

(some working or explanation must be given for 2 marks)

1(d) $v = -3\cos(2\pi t)$

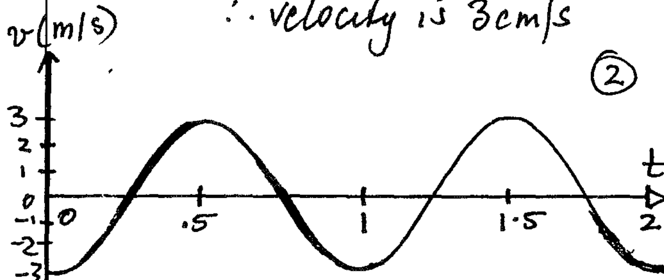
(i) $t=0.5 \quad v=-3\cos(2\pi\times.5$

$= -3\cos\pi$

$= 3$ ①

$\therefore$ velocity is 3cm/s

②



(iii) direction changes when sign

v changes sign

$\therefore$ 4 times ①

(2) (a)(i) $P(2) = \overset{①}{3}\times(.2)\overset{①}{(.2)}(.8)$

$= 0.096 = 9.6\%$

(ii) $P(\text{at least one}) = 1-P(\text{none})$

$\therefore \frac{99.9}{100} > 1-(.8)^n$ ①

$\therefore (0.8)^n < 1-0.999$

$\therefore (0.8)^n < 1\times10^{-3}$

$\therefore n > \frac{\log(1\times10^{-3})}{\log(0.8)}$ ①

$n > 30.95655348$ ①

$\therefore$ plant 31 seeds

(b) (i) $(x-l)(x-m)-9n^2$

$= x^2-(l+m)x+lm-9n^2$

$\Delta = b^2-4ac$

$\Delta = (l+m)^2-4(1)(lm-9n$

$\Delta = l^2+2lm+m^2-4lm+36$

$\Delta = l^2-2lm+m^2+36n^2$

(ii) $\Delta = (l-m)^2+36n^2$ ①

both terms are perfect squares

$\therefore \Delta \geq 0$ ①

$\therefore$ equation has real roots

2 (c) (i) $\ddot{x} = \frac{5}{(t+1)^2}$

$t = 0 \quad \dot{x} = 5$

$\dot{x} = \int \frac{5}{(t+1)^2} dt$

$= \frac{-5}{(t+1)} + c$ ①

$t = 0 \quad \dot{x} = 5 \quad c = 10$

$\therefore \dot{x} = \frac{-5}{(t+1)} + 10$ ①

[Alternative solution
show $\dot{x} = 5$ when $t = 0$ ①
and show $\frac{d\dot{x}}{dt} = \ddot{x}$] ①

(ii) for stationary $\dot{x} = 0$

$\therefore 0 = \frac{-5}{t+1} + 10$

$\therefore t = -\frac{1}{2}$ ①

but $t \geq 0 \quad \therefore \dot{x} \neq 0$

$\therefore$ never stationary.

[Alternative solution
$\dot{x} = \frac{-5}{t+1} + 10$
for $t \geq 0 \quad \frac{-5}{t+1} > -10$
$\therefore \dot{x} > 0 \quad \therefore$ never stationiary] ①

(iii) $\dot{x} = \frac{-5}{t+1} + 10$

$x = \int (\frac{-5}{t+1} + 10) dt$ ①

$\therefore x = -5\ln(t+1) + 10t + k$

$t = 0 \quad x = 0 \quad \therefore k = 0$

$x = 10t - 5\ln(t+1)$ ①

(iv) The particle moves in a positive direction from the origin, at 5m/s and continues to accelerate to a velocity ~~of~~ approaching 10m/s as time increases.

(minimum answer: particle accelerates in positive direction fro
origin.) ①

3(a) (i) $10000e^{0.2t} = 15000e^{0.15t}$ ($\frac{1}{2}$)

$\therefore \frac{10}{15} = \frac{e^{0.15t}}{e^{0.2t}}$

$\therefore \frac{10}{15} = e^{-0.05t}$ ($\frac{1}{2}$)

$t = \frac{\ln(\frac{10}{15})}{-0.05t}$

$t = 8.109302162$ ($\frac{1}{2}$)

$\therefore$ equal population after
8.1 years from 1 Jan 200
$\therefore$ Populations equal durin
2008 ($\frac{1}{2}$)

(ii)

$\frac{dP_A}{dt} = 0.2 \times 10000e^{0.2t}$ ①

$t = 8.109302162\cdots$

$= 10125$ people/year

(Accept ±1 person)

(b) (i)

Die ①

Die ②

	1	2	3	4	5	6
1	4	5	6	7	8	9
2	5	6	7	8	9	10
3	6	7	8	9	10	11
6	7	8	9	10	11	12
6	7	8	9	10	11	12
6	7	8	9	10	11	12

①

(ii) (α) $P(11) = \frac{4}{36} = \frac{1}{9}$

(β) $P(\text{Less than } 9) = \frac{18}{36} = \frac{1}{2}$

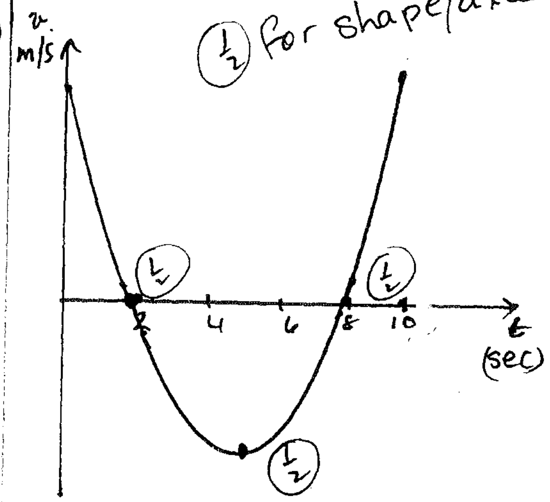
(iii) $\left.\begin{array}{l} P(>10) \\ \text{Normal Die} \end{array}\right\} = \frac{3}{36}$ ①

$\left.\begin{array}{l} P(>10) \\ \text{Other Die} \end{array}\right\} \frac{15}{36}$

$P(>10) = \frac{1}{2} \times \frac{3}{36} + \frac{1}{2} \times \frac{15}{36} = \frac{1}{4}$

[some working or explanation must be given for full marks]

3(c)



(½) for shape/axes

(ii)(α) Rest $\dot{x} = 0$

$t = 2$ and $t = 8$ sec
(½) (½)

(β) $\ddot{x} = 0$ $t = 5$ sec ①

(γ) $\dot{x} < 0$ $2 < t < 8$ sec.

(must have < signs)

(δ) $t > 4 \cdot 6$ sec ①

(Accept ± 0.2 for all answers)

(4) (a) $2(x-3)(x-\pi) > 0$

for $2(x-3)(x-\pi) = 0$

$x = 3$ or $x = \pi$ ①

$\therefore$ $x < 3$ or $x > \pi$ ①

(b) (i) $N = N_0 e^{kt}$

$$\frac{dN}{dt} = k\left(N_0 e^{kt}\right)$$

$$= kN$$ ①

$$\therefore \frac{dN}{dt} \propto N$$

(ii) $700000 = 500000\, e^{2k}$

$2k = \ln\left(\frac{7}{5}\right)$

$\therefore$ $k = \frac{1}{2}\ln\left(\frac{7}{5}\right)$ ①

$250000 = 500000\, e^{\frac{1}{2}\ln\left(\frac{7}{5}\right)t}$

$\therefore \frac{1}{2} = e^{\frac{1}{2}\ln(7/5)t}$

$\therefore t = \dfrac{\ln\frac{1}{2}}{\frac{1}{2}\ln\left(\frac{7}{5}\right)}$ ①

$\therefore t = -4 \cdot 12$ hours (½)

= 4 hours 7 mins before 3pm

$\therefore$ time was 10:53 am (½)

(c) (i) $x = 2\cos t + t\sin t$ ①

$\dot{x} = -2\sin t + t\cos t + \sin t$

$\dot{x} = -\sin t + t\cos t$ ①

$= t\cos t - \sin t$

(ii) Rest $\dot{x} = 0$

$\sin t = t\cos t$

$\therefore t = \dfrac{\sin t}{\cos t}$

$\therefore t = \tan t$ ①

(iii) $x = 2\cos t + t\sin t$ ①

$= 2\cos t + \tan t \sin t$

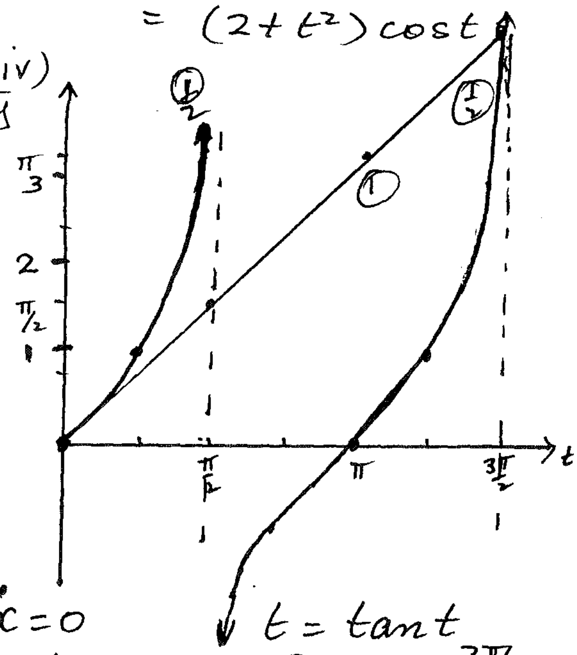
$= 2\cos t + \tan^2 t \cos t$

$= \cos t\left[2 + \tan^2 t\right]$

$= \cos t\left[2 + t^2\right]$ ①

$= (2 + t^2)\cos t$

(iv)



(v) $\dot{x} = 0$ $t = \tan t$

$\therefore t = 0$ or $\pi/2 < t < 3\pi/2$

① $\therefore x = 2$ or $x = (2+t^2)\cos t$

but $\cos t < 0$ for $\pi/2 < t < 3\pi/2$

$\therefore (2+t^2)\cos t < 0$ ①

for 2nd solution of t.

$\therefore$ particle is at rest on opposite sides of the origin