JRAHS 2005 TRIAL HSC – MATHEMATICS (2 Unit)

Question 1. **[Start a New Page]** **Marks**

(a) Evaluate $\dfrac{8\pi}{2+\sqrt[3]{2}}$ as a decimal correct to 3 significant figures. **1**

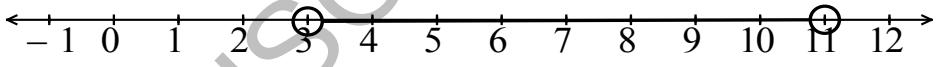
(b) Find $\dfrac{d}{dx}[5x+\tan x]$. **2**

(c) A group of cards are labelled 0, 1, 2, … and 20. **2**
Find the probability that when a card is chosen, it is a prime number?

(d) Find $\int \sec 3x \tan 3x\, dx$, using the table of standard integrals. **2**

(e) Find the exact value for $\sec\dfrac{\pi}{6}$. **1**

(f) The number line graph represents the solutions to the inequality equation $|x-a| \le b$. **2**



Find the values of a and b.

(g) Find integer p so that $(3-\sqrt{5})^2 = 14-\sqrt{p}$. **2**

Question 2. **[Start a New Page]**

(a) Differentiate with respect to x the following:

(i) $\dfrac{\sin x}{x+1}$. **2**

(ii) $\sqrt{1+e^{6x}}$. **2**

(iii) $x^3 \ln x$. **2**

(b) Find the domain for the function: $y = x + \ln(3 - x)$. **1**

(c) Solve for θ: $\tan\theta = 0 \cdot 3$ (correct to two decimal places) for $0 < \theta < 2\pi$. **2**

(d) Find a primitive function for $\dfrac{1}{3x}$. **1**

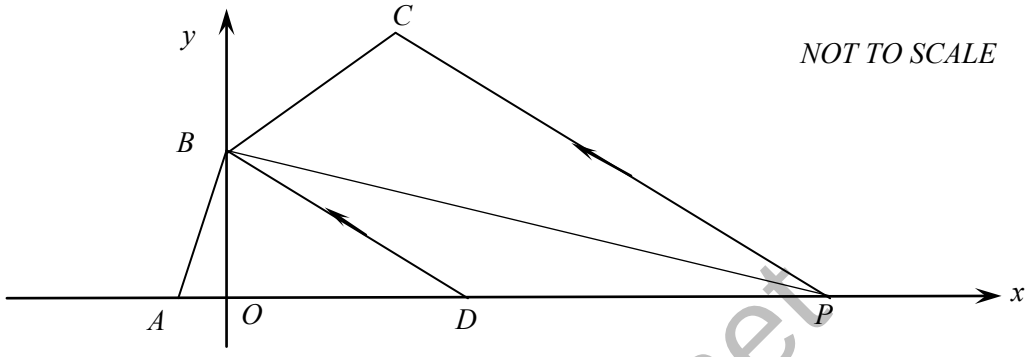
(e) Evaluate $\int_0^2 (e^{-x} + 1)\,dx$, to two decimal places. **2**

Question 3. **[Start a New Page]** **Marks**

(a) Find the equation of the normal to the curve $y = 2\cos x + 3$ **2**

at the point $(\frac{\pi}{2}, 3)$.

(b) The diagram shows the coordinates of four points:
$A\ (-1, 0)$, $B(0, 2)$, $C(3, 5)$ and $D(8, 0)$.



(i) Find the gradient of BD. **1**

(ii) Find the equation of the line passing through B and D. **1**

(iii) Find the angle of inclination of the line BD (to nearest the degree). **1**

(iv) Given $CP \parallel BD$, show that the equation of CP is $x + 4y - 23 = 0$. **2**

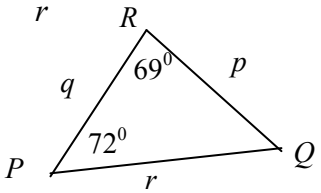
(v) Find the coordinates of point P, where the line CP intersects the x-axis. **1**

(vi) Find the perpendicular distance of point C from the line segment BD. **2**

(vii) Explain why the area of quadrilateral $ABCD$ is the same as the area of triangle ABP. **2**

Question 4. **[Start a New Page]** **Marks**

(a) For ΔPQR, find $\frac{p}{r}$ correct to 3 decimal places. **2**



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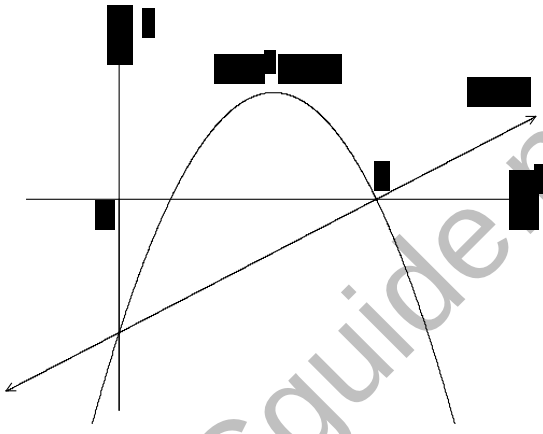
(b) The roots of the equation $x^2 - 2x - 5 = 0$, are $x = \alpha$ and $x = \beta$. **2**

Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

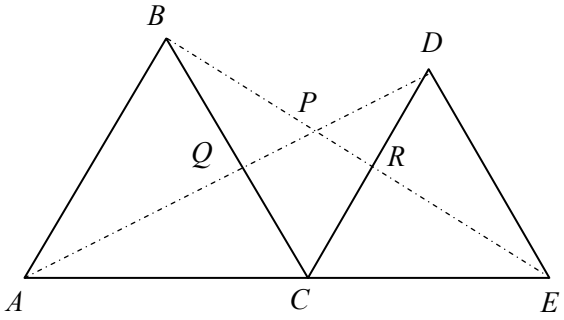
(c) The diagram shows the sketch of the parabola $y = 6x - x^2 - 5$ and a line $y = x - 5$.



(i) Find the x-coordinate of A. **1**

(ii) Find the area of the shaded region bounded by the line and the parabola. **2**

(d) Given the triangles ABC and CDE are different equilateral triangles. BE intersects AD at P, and A, C and E lie on the line segment AE.



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(i) *Copy* the diagram onto your writing booklet, and prove that $\Delta ACD \equiv \Delta ECB$. **3**

(ii) Show that $\angle APB = 60^0$. **2**

Question 5. **[Start a New Page]** **Marks**

(a) Simplify: $1+\cos^2 x+\cos^4 x+\ldots$ for $0<x<\frac{\pi}{2}$. **1**

(b) The first term of an Arithmetic series is a, the common difference is d and the n^{th} term is L.

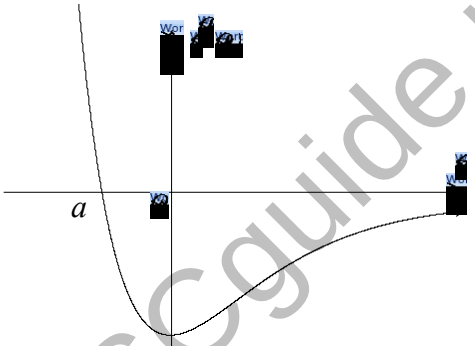
(i) Write down L, in terms of a, d and n. **1**

(ii) Show that the sum, S_n, of the first n terms can be expressed as **2**

$$S_n=\frac{(L+a)}{2}\left[1+\frac{L-a}{d}\right].$$

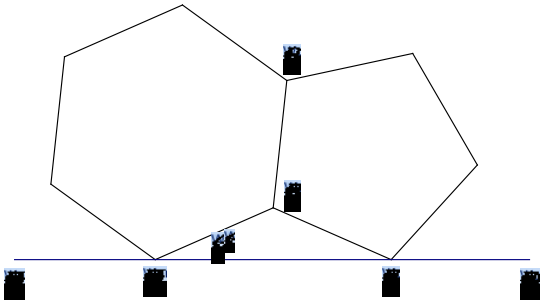
(iii) Hence, or otherwise, find: $5+8+11+\ldots+173$. **1**

(c) The sketch of the Gradient function $f'(x)$ is shown below. **3**



Sketch the graph of the function, $y=f(x)$, given $f(x)>2$ for $x>0$.

(d) The figure consists of a regular hexagon and a regular pentagon, with a common side AB. **4**



Given vertex H and P lie on a straight line $XHPY$ and $\angle PHA=x^0$.

Copy the diagram onto your writing booklet and find the value of x. Give reasons.

Question 6. **[Start a New Page]** **Marks**

(a) Solve for x: $\log_3(2x-5)=1$. **2**

(b) Differentiate $f(x)=\frac{3}{x}$ with respect to x, by first principles. **2**

(c) A particle is moving in a straight line with velocity $v=3e^{t}+6e^{-t}$.
It begins its motion at the Origin O, t is in seconds and v is in metres per second.

(i) What is its initial velocity? **1**

(ii) Is the particle ever at rest? Give reasons **1**

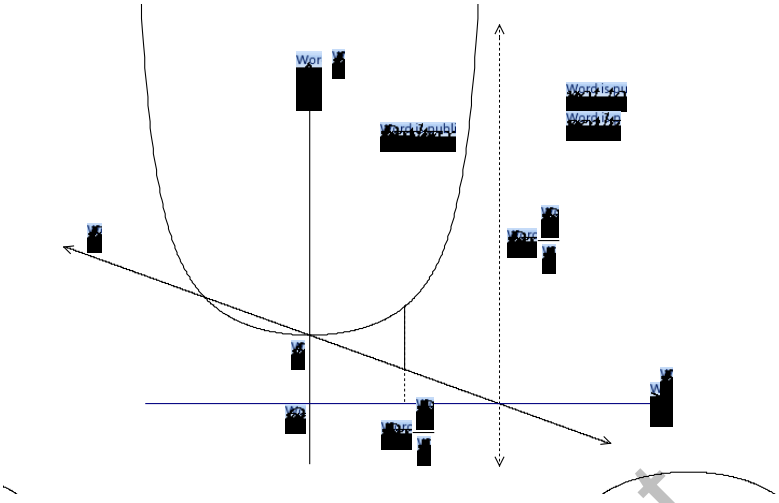
(iii) Find the displacement function, x, of the particle, at time t minutes. **2**

(iv) Find the time when the particle is at $x=10$. **2**

(d) Sketch the graph of the curve $y=3\sin 2x$ for the interval $0\le x\le\pi$. **2**

Question 7. **[Start a New Page]** **Marks**

(a) Given the sketch of the curve $y = \sec x$, for $0 \le x < \frac{\pi}{2}$ and the line L as shown.



(i) Show that the equation of the line, L, is $y = 1 - \frac{2}{\pi}x$. **1**

(ii) The area enclosed by the curve $y = \sec x$, the lines L and $x = \frac{\pi}{4}$, **3**
is rotated about the x-axis.
Find the exact volume for the solid of revolution.

(b) Use Simpson's rule, with three function values, to evaluate : $\int_0^4 \frac{3dx}{1+\sqrt{x}}$, **2**

(correct to two decimal places).

(c) Consider the function: $f(x) = x^3 - 3kx + 4$.

(i) Explain why $f(x)$ is an increasing function for all x when $k < 0$. **2**

(ii) Find the expression for each stationary point of $f(x)$, **2**
in terms of k, when $k > 0$.

(iii) Prove that $f(x)$ has 3 distinct real x-intercepts when $k^3 > 4$. **2**

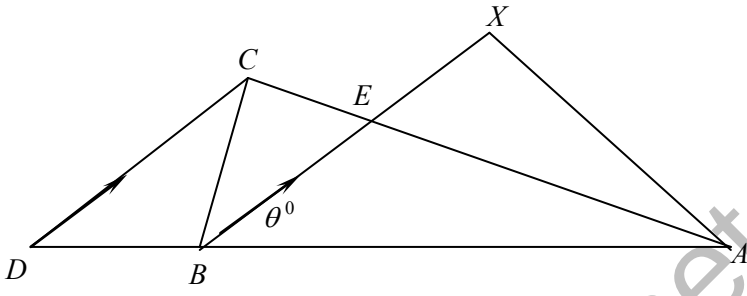
Question 8. **[Start a New Page]** **Marks**

(a) (i) Show that the locus of all points $P(x, y)$, which are equidistant from the origin O and to the line $y = 4$, is the parabola: **2**

$$x^2 = 16 - 8y.$$

(ii) Hence, find the coordinates of the vertex V. **1**

(b) In the diagram $BX \parallel DC$, XB bisects angle ABC and $AX \perp BX$ at X.
Let $\angle ABX = \theta^0$.



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Copy this diagram onto your writing booklet.

(i) State why $\angle BCD = \angle XBC$. **1**

(ii) Show that triangle BCD is isosceles. **2**

(iii) Hence, explain why $\frac{AE}{EC} = \frac{AB}{BC}$. **2**

(iv) If $\frac{BA}{BC} = 3$, and by using the cosine rule, or otherwise, **4**
show that E is the midpoint of BX.

Question 9. **[Start a New Page]** **Marks**

(a) The mass of a substance X is M grams, at time t years.
It decays at an instantaneous rate proportional to its mass at time t years,
ie $\frac{dM}{dt} = -kM$, where k is the decay rate constant of proportionality.

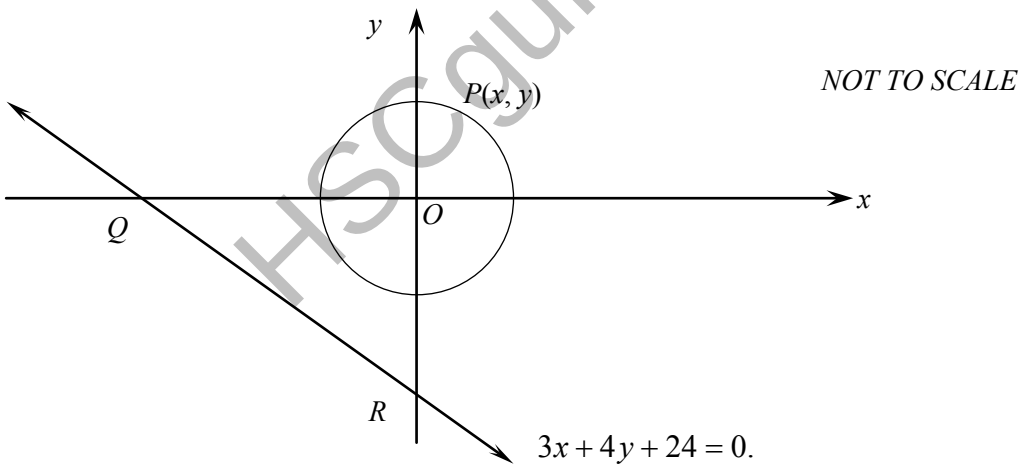
(i) Verify that $M = M_0 e^{-kt}$ satisfies the rate equation $\frac{dM}{dt} = -kM$, **1**
where M_0 is the initial mass of substance X.

(ii) Hence, show that the time, T years, for half the mass of X to decay, **1**
is given by $T = \frac{\ln 2}{k}$.

(iii) Find the decay rate constant, if the half-life of substance X is 3 466 years? **1**

(iv) Sketch the graph of $\frac{dM}{dt}$ against M. **1**

(b) Consider the circle: $x^2 + y^2 = 1$ and the line: $3x + 4y + 24 = 0$.
The points Q and R are the x and y-intercepts of the line $3x + 4y + 24 = 0$.
The point $P(x, y)$ lies on the circle as shown in the diagram.



(i) As P moves around the circle, show that the perpendicular distance, W, **3**
from the length QR to the circle, is given by: $W = 3x + 4y + 24$.

(ii) Hence, or otherwise, find the least length of W. **5**

Question 10. **[Start a New Page]** **Marks**

(a) Mary visits the sock section of a shop that has 5 different pairs of socks individually arranged on a table.
She randomly selects socks one at a time.

(i) Explain why the probability that Mary does *not* have a matching pair of socks, after selecting the second sock, is $\frac{8}{9}$. **1**

(ii) Find the probability she does *not* have a matching pair of socks after selecting the third sock. **2**

(iii) What is the probability that, in the first 3 socks, Mary does have a matching pair? **1**

(b) Mr Howzat borrows \$30 000 from a bank. Interest is to be calculated at 12% *pa*, compounded monthly, on the balance remaining over the term of the loan of 7 years.
Each year, at k regular intervals, (where $k = 1, 2, 3, \ldots$ or 12), Mr Howzat repays \$$F$ for each instalment.

(i) Show that the amount owing, \$$A_2$, after the second instalment is paid, is given by: **2**

$$A_2 = 30\,000\left[(1\cdot01)^{\frac{12}{k}}\right]^2 - F[1+(1\cdot01)^{\frac{12}{k}}].$$

(ii) Show that the amount of each instalment, \$$F$, is given by: **3**

$$F = 30\,000\times1\cdot01^{84}\times\frac{[1\cdot01^{\frac{12}{k}}-1]}{[1\cdot01^{84}-1]}.$$

(iii) Calculate the value of each instalment if the instalments are made quarterly ($k = 4$). **1**

(iv) How much would Mr Howzat have saved over the term of the loan if he had chosen to make monthly rather than quarterly instalments? **2**

THE END

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