JRAHS Year 12 Term 1 Mathematics (2 Unit) Assessment – 2006

SOLUTIONS

Question 1 (15 Marks) Marks

(a) Integrate with respect to x:

(i) $\frac{1}{3}x^3 - \frac{2}{3}x\sqrt{x} + c$	each part = 1 mark - 1 for no "c" only once in part (a)	**2**
(ii) $-\frac{1}{8}\cos 8x + c$	$\cos 8x$ = 1 mark -1/8 …… = 1 mark	**2**
(iii) $-2\ln(1-3x) + c$ or $-2\ln\lvert 1-3x\rvert + c$	ln(1-3x) = 1 mark -2 ….. = 1 mark	**2**

(b) Evaluate:

(i) $\int_0^3 e^{2x+1}\,dx = \left[\frac{1}{2}e^{2x+1}\right]_0^3$	primitive = 1 mark	**2**
$= \frac{1}{2}\left(e^7 - e^1\right)$	evaluation = 1 mark	
(ii) $\int_0^{\pi} \cos\left(\frac{1}{4}x\right)dx = \left[4\sin\left(\frac{1}{4}x\right)\right]_0^{\pi}$	primitive = 1 mark	**2**
$= 4\left(\sin\left(\frac{\pi}{4}\right) - \sin 0\right)$ $= 4 \times \frac{1}{\sqrt{2}} - 0$ $= 2\sqrt{2}$	evaluation = 1 mark	
(iii) $\int_1^4 \frac{2x^2-3}{x}\,dx = \int_1^4 \left(2x - \frac{3}{x}\right)dx$		**2**
$= \left[x^2 - 3\ln x\right]_1^4$	primitive = 1 mark	
$= (16 - 3\ln 4) - (1 - 3\ln 1)$ $= 15 - 3\ln 4$	evaluation = 1 mark	

(c) (i) $\sum_{r=1}^{5} r^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$ $= 55$	accept correct answer with no working	**1**
(ii) $n = 200, \quad a = 3, \quad d = 4$		**2**
$S_{200} = \frac{200}{2}\{2 \times 3 + 199 \times 4\}$	sub. into correct formula = 1 mark	
$= 80200$	correct sum = 1 mark	

Question 2 (15 Marks) Marks

(a) Find:

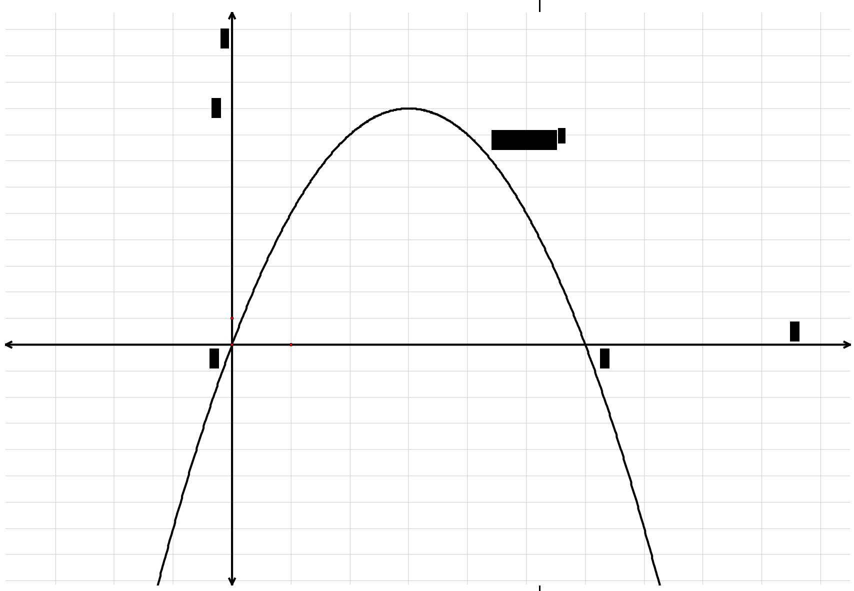
(i) $\int (3x-2)^8 \, dx = \frac{1}{27}(3x-2)^9 + c$

$(3x-2)^9$ = 1 mark
no penalty for missing “c” **2**

(ii) $\int \frac{e^{3x}}{5+e^{3x}} \, dx = \frac{1}{3}\ln\left(5+e^{3x}\right) + c$

$\ln\left(5+e^{3x}\right)$ = 1 mark
no penalty for missing “c” **2**

(b) (i) **2**



both intercepts = 1 mark
shape = 1 mark

(ii)
$A = \int_0^6 \left(6x - x^2\right) dx$

$= \left[3x^2 - \frac{1}{3}x^3\right]_0^6$

$= \left(3\times 6^2 - \frac{1}{3}\times 6^3\right) - (0-0)$

$= 36$

$\therefore$ Area $= 36\, u^2$

correct integral = 1 mark **3**

integration = 1 mark

evaluation = 1 mark

max. 2/3 if no units

(c) In ΔABC, $AP \perp BC$ and $BQ \perp AC$.

Solution	Marking criteria	Marks
(i) In $\Delta AQR \parallel\!| \Delta BPR$ $A\hat{Q}R = B\hat{P}R$ (both 90°) $A\hat{R}Q = B\hat{R}P$ (vertically opposite angles are equal) $\therefore \Delta A\hat{Q}R \parallel\!| \Delta B\hat{P}R$ (equiangular)	angle pairs (= 1 mark each) = 2 marks test = 1 mark	**3**
(ii) Area $\Delta ABR = \dfrac{BR \times AQ}{2}$ $\dfrac{BP}{12} = \dfrac{6}{9}$ (ratio of corresponding sides in similar triangles are equal) $BP = 8$ $BR^2 = 8^2 + 6^2$ (Pythagoras' Theorem) $BR = 10$ Area $= \dfrac{10 \times 12}{2} u^2$ $= 60\ u^2$	calculating 2 needed lengths (= 1 mark each) = 2 marks several pairs of sides and various triangle combinations can be used to find the required area area = 1 mark	**3**

Question 3 (15 Marks) Marks

(a) $V = \pi \int_0^2 x^2 \, dy$ where $y = \sqrt{4-x}$ **3**

$y^2 = 4 - x$

$x = 4 - y^2$

$V = \pi \int_0^2 \left(4 - y^2\right)^2 \, dy$ — integral = 1 mark

$= \pi \int_0^2 \left(16 - 8y^2 + y^4\right) dy$ — wrong intergrand = max 1/3

$= \pi \left[16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\right]_0^2$ — integration = 1 mark

$= \pi \left\{\left(16 \times 2 - \frac{8}{3} \times 8 + \frac{1}{5} \times 32\right) - 0\right\}$

$= \frac{256\pi}{15}$

$\therefore \; Volume = \frac{256\pi}{15} \, u^3$ — evaluation = 1 mark

no penalty for missing units

(b) (i) when $x = \frac{\pi}{6}$ $\cos x = \cos \frac{\pi}{6}$ **2**

$= \frac{\sqrt{3}}{2}$

$\sin 2x = \sin \frac{2\pi}{6}$

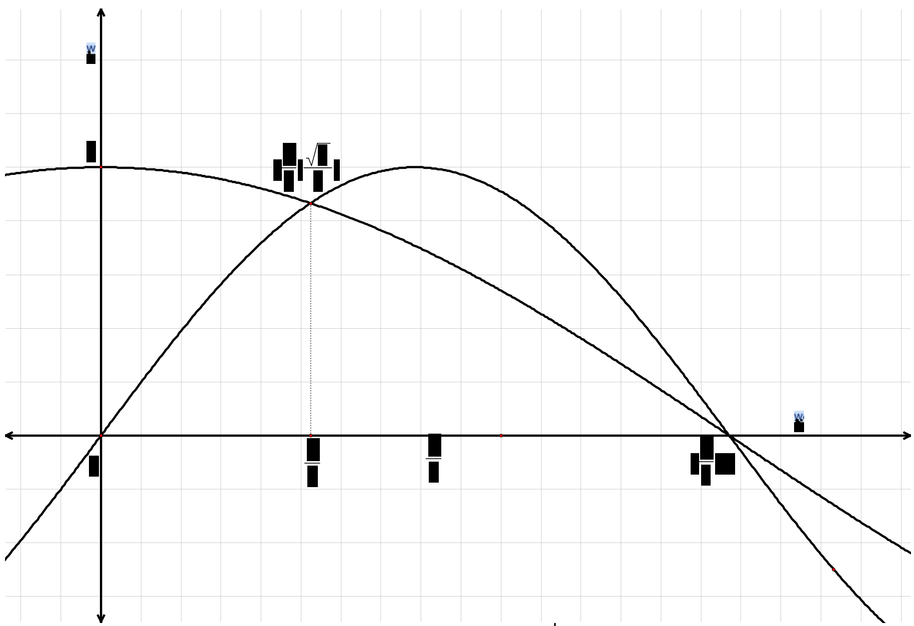
$= \sin \frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

$\therefore$ both curves meet at $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$

show that each y value = $\frac{\sqrt{3}}{2}$
(= 1 mark each) = 2 marks

(ii) **2**



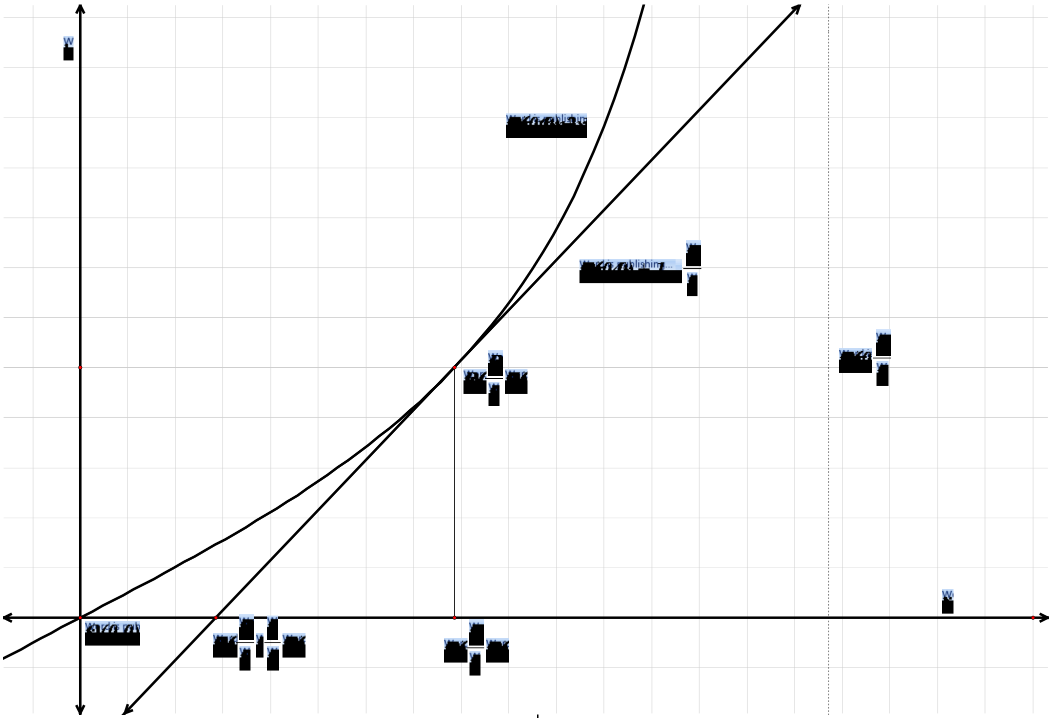
1 mark for each curve
no penalty if domain extends beyond $\frac{\pi}{2}$
no penalty for missing intersection point

		Solution	Marking criteria	Marks
	(iii)	$A = \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\, dx$	integral = 1 mark	**3**
		$= \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$	integration = 1 mark	
		$= \left(-\frac{1}{2}\cos\pi - \sin\left(\frac{\pi}{2}\right)\right) - \left(-\frac{1}{2}\cos\left(\frac{2\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right)$		
		$= \frac{1}{4}$	evaluation = 1 mark no penalty for missing units	
		$\therefore$ Area $= \frac{1}{4} u^2$		
(c)	(i)	Let value of fund at end of n^{th} month = $\$A_n$		**2**
		$A_1 = 500 \times 1.005$		
		$A_2 = (A_1 \times 1.005 + 500) \times 1.005$		
		$= 500(1.005^2 + 1.005)$		
		$\vdots$		
		$\vdots$		
		$A_{12} = 500(1.005^{12} + 1.005^{11} + ... + 1.005)$	correct expression = 1 mark	
		$= 500 \times 1.005 \dfrac{(1.005^{12} - 1)}{1.005 - 1}$		
		$= 6167.78$	evaluation of sum = 1 mark	
		Value $= \$6167.78$		
	(ii)	$A_n = 500(1.005^n + 1.005^{n-1} + ... + 1.005)$	correct general expression = 1 mark	**3**
		$= 500 \times 1.005 \dfrac{(1.005^n - 1)}{0.005}$		
		$= 100500(1.005^n - 1)$		
		$\therefore\ 20000 = 100500(1.005^n - 1)$		
		$1.005^n = \frac{241}{201}$	simplified equation = 1 mark	
		$n \ln(1.005) = \ln\left(\frac{241}{201}\right)$		
		$n = \dfrac{\ln\left(\frac{241}{201}\right)}{\ln(1.005)}$		
		≈ 36.39	solve equation = 1 mark (by trial and error or logs) accept 37 months	
		no. months $= 36$		

Question 4 (15 Marks) Marks

(a)	(i)	circumferences: $\{80\pi, 100\pi, 120\pi, ...\}$ $a = 80\pi, d = 20\pi$ $T_{10} = 80\pi + 9 \times 20\pi$ $= 260\pi$ Length $= 260\pi\ cm$ $\approx 816.81\ cm$ $= 8m$ (to nearest metre)	indication of correct "*a*" and "*d*" = 1 mark evaluate length = 1 mark (using their "*a*" and "*d*") correct decimal approx. = 1 mark	**2**
	(ii)	$S_n = \frac{n}{2}\{2 \times 80\pi + (n-1) \times 20\pi\}$ when $S_n = 200000$ $\frac{n}{2}\{2 \times 80\pi + (n-1) \times 20\pi\} = 200000$ $\pi n^2 + 7\pi n - 20000 = 0$ $n = \frac{-7\pi \pm \sqrt{80049\pi}}{2\pi}$ $n \approx -83.2$ or 76.3 but $n > 0$ $\therefore$ no. complete circles = 76	sum formula with substitution = 1 mark simplified quadratic equation = 1 mark correct approximation = 1 mark	**3**
(b)	(i)	$f(x) = \ln(\cos 2x)$ $f'(x) = \frac{-2\sin 2x}{\cos 2x}$ $= -2\tan 2x$		**1**
	(ii)	$y = \tan 2x$ $y' = 2\sec^2 2x$ when $x = \frac{\pi}{8}$ $\quad y' = 2\sec^2\left(\frac{\pi}{4}\right)$ $= 2\left(\sqrt{2}\right)^2$ $= 4$ tangent is: $y - 1 = 4\left(x - \frac{\pi}{8}\right)$ $y - 1 = 4x - \frac{\pi}{2}$ $y = 4x + 1 - \frac{\pi}{2}$	derivative = 1 mark gradient = 1 mark tangent equation = 1 mark	**3**
	(iii)	at Q, $\quad y = 0$ $\therefore\ 4x + 1 - \frac{\pi}{2} = 0$ $x = \frac{\pi}{8} - \frac{1}{4}$ Q is $\left(\frac{\pi}{8} - \frac{1}{4}, 0\right)$		**1**

(iv) **2**



tan$2x$ graph = 1 mark

graph of tangent line = 1 mark

no penalty if domain extends beyond $\frac{\pi}{4}$

(v) **3**

$$A = \int_0^{\pi/8} \tan 2x \, dx - \frac{QR \times PR}{2}$$

$$= \int_0^{\pi/8} \tan 2x \, dx - \frac{\frac{1}{4} \times 1}{2}$$

$$= \left[-\tfrac{1}{2}\ln(\cos 2x)\right]_0^{\pi/8} - \tfrac{1}{8}$$

$$= -\tfrac{1}{2}\left\{\ln\left(\cos\tfrac{\pi}{4}\right) - \ln(\cos 0)\right\} - \tfrac{1}{8}$$

$$= -\tfrac{1}{2}\left\{\ln\left(\tfrac{1}{\sqrt{2}}\right) - \ln(1)\right\} - \tfrac{1}{8}$$

$$= -\tfrac{1}{2}\left\{-\tfrac{1}{2}\ln 2 - 0\right\} - \tfrac{1}{8}$$

$$= -\tfrac{1}{8} + \tfrac{1}{4}\ln 2$$

$$\therefore \text{ Area} = \left(-\tfrac{1}{8} + \tfrac{1}{4}\ln 2\right)u^2$$

area of triangle = 1 mark

$\left[-\tfrac{1}{2}\ln(\cos 2x)\right]_0^{\pi/8}$ = 1 mark

correct values for "a" and "b" = 1 mark

This is the end of the Examination Paper