

QUESTION 1 (15 Marks)**Marks**

- (a) By “completing the square” or otherwise, find the minimum value of $2x^2 - 4x + 5$ for all real values of x . 2
- (b) A radioactive substance of mass M decays at a rate proportional to its mass present, ie; $M = M_0 e^{kt}$.
Its initial mass is 400 grams and reduces to 300 grams after 2 years.
- (i) How many grams have decayed at the end of four years? 4
- (ii) The half-life of the substance is the time taken for the mass to be reduced by half. What is the half-life of the substance, (to the nearest month). 3
- (c) The equation of motion of an object moving x cm about a fixed point O , after t seconds, along a straight line is given by $x = \tan t$ for $0 \leq t < \frac{\pi}{2}$.
- (i) Express its velocity $v(t)$ and acceleration $a(t)$ in terms of t . 3
- (ii) Show that its acceleration is $a(t) = 2x(1+x^2)$. 3

QUESTION 2 (15 Marks)

- (a) For what values of k is $kx^2 + (3+k)x + (3+k)$ positive definite. 4
- (b) Nine marbles numbered 1, 2, 3, 4, 5, 6, 7, 8 & 9 are placed in a bag and three are drawn out at random (each not replaced). What is the probability that the sum of the numbers on the three marbles drawn is odd. 3
- (c) The equation of motion of a particle moving x metres along a straight line after t seconds is given by $x(t) = 2t^3 - 6t^2 + 3$.
- (i) What is the particle’s initial speed and acceleration. 2
- (ii) When and where does it first come to rest. 2
- (iii) Sketch a velocity/time graph and briefly explain what is happening to the motion when $t = 1$ second. 2
- (iv) Briefly describe the motion of the particle 2

QUESTION 3 (15 Marks)**Marks**

- (a) The population of two colonies after t years is given by $P_1 = 2000e^{0.138t}$ and $P_2 = 5000e^{0.04t}$. The initial population of each was recorded on the 1st January, 2006.
- (i) How long will it take for the population P_1 to triple. **2**
(Answer to the nearest year.)
- (ii) Calculate the year and month when both populations are the same. **3**
- (iii) Calculate the rate at which P_1 is increasing at this time. **2**
- (b) Two chess players, Bostik and Spastik, play three games of chess to contest a win. In any game they play, the probability that Bostik wins is $\frac{5}{10}$ and the probability that Spastik wins is $\frac{4}{10}$.
What is the probability that Bostik wins the competition. **3**
- (c) An object moves x cm along a straight line after t seconds with its velocity function $v = -e^{-2t}$ for $t \geq 0$ and is initially at the origin.
- (i) Derive an expression for its displacement as a function of time. **3**
- (ii) Neatly sketch $x = f(t)$ for $t \geq 0$. **2**

QUESTION 4 (15 Marks)

- (a) Find all real values of t for which the quadratic equation $\frac{x^2 - x + 1}{x^2 + x + 1} = t$ has real and different roots. **4**
- (b) A particle of unit mass is projected vertically upwards from a point O with a velocity of 25 m/s and has an acceleration of -10 m/s².
- (i) Find its velocity and height above O after a time $t > 0$. **3**
- (ii) Find its maximum height of projection. **2**
- (b) A deck of cards contains the red Jacks, Queens, Kings and Aces from a normal pack of 52 playing cards (ie; the Hearts and Diamonds). Jenny is dealt two cards from this deck of cards. What is the probability that:
- (i) She has two Aces if she announces that she has at least one Ace. **2**
- (ii) She has a pair if she announces that she does not have the Ace of Hearts. **2**
- (iii) She has two cards of the same suit if she announces that she has at least one Heart and one King. **2**

END of PAPER