YEAR 12 TERM 2
H.S.C. ASSESSMENT 2006
SOLUTIONS

1 (a) $2x^2 - 4x + 5$

$= 2\left(x^2 - 2x + \frac{5}{2}\right)$

$= 2\left(x^2 - 2x + 1 + \frac{3}{2}\right)$

$= 2(x-1)^2 + 3$

$\therefore$ MIN of 3 at $x = 1$

OR using $x = -\frac{b}{2a}$

$x = 1 \quad \therefore 2(1)^2 - 4(1) + 5$

$= 3$

(b)(i) $M = M_0 e^{kt}$

$= 400 e^{kt}$

When $t = 2$, $M = 300$

$\therefore 300 = 400 e^{2 \cdot k} \quad \therefore k = \frac{1}{2} \ln \frac{3}{4}$

When $t = 4$

$M = 400 e^{\frac{4}{2} \ln \frac{3}{4}}$

$= 400 \left(\frac{3}{4}\right)^2$

$= 225$

$\therefore$ 175 gms. decayed.

(ii) When $M = \frac{1}{2} M_0$

$\therefore \frac{1}{2} M_0 = M_0 e^{\frac{t}{2} \ln \frac{3}{4}}$

$\therefore \ln \frac{1}{2} = \frac{t}{2} \ln \frac{3}{4}$

$\therefore \quad t = \frac{2 \ln 0.5}{\ln 0.75}$

$\therefore \quad t = 4.82$ years

$= 4$ years 10 months.

(c) (i) $x = \tan t$

$\therefore v(t) = \sec^2 t \quad -(1)$

$\therefore a(t) = 2 \sec t \cdot \sec t \tan t$

$\therefore a(t) = 2 \sec^2 t \tan t \quad -(2)$

(ii) $a(t) = 2 \tan t \,(1 + \tan^2 t)$

$\therefore a(t) = 2x(1 + x^2)$

since $x = \tan t$

2 (a) $kx^2 + (3+k)x + (3+k)$

We require $k > 0$ and $\Delta < 0$

$\Delta = (3+k)^2 - 4k(3+k)$

$= (3+k)(3+k-4k)$

$= (3+k)(3-3k)$

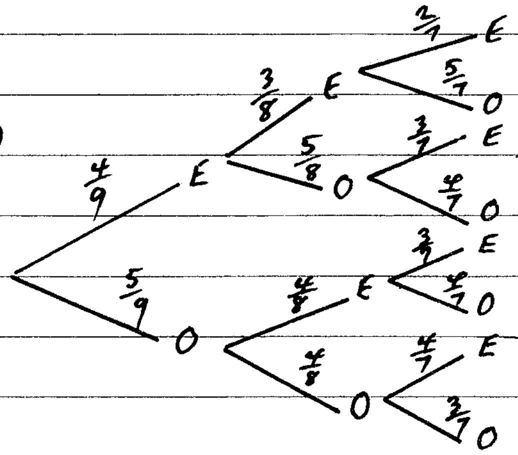
$= 3(3+k)(1-k)$

For $\Delta < 0$

$-3 \qquad 1$

For $k > 0$ we require only $k > 1$.

(b)



$P = P(E+E+O) + P(O+O+O)$

$= 4\left(\frac{4}{9} \times \frac{3}{8} \times \frac{5}{7}\right)$

$= \frac{4 \times 5}{3 \times 2 \times 7}$

$\therefore P = \frac{10}{21}$

2 (c)(i) $x(t) = 2t^3 - 6t^2 + 3$

$v(t) = 6t^2 - 12t$

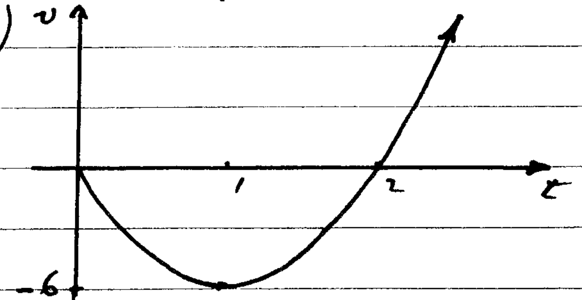
$a(t) = 12t - 12$

When $t = 0$

$v(0) = 0$ m/s and $a(0) = -12$ m/s.

(ii) It comes to rest when $v = 0$ ie; when $t = 0$ or $t = 2$. Since $t > 0$ $\therefore$ when $t = 2$ and $x = -5$ ie; 5 metres left of 0.

(iii)



When $t = 1$ it reaches maximum speed OR zero force acting on the particle

(iv) The particle starts 3 m right of 0 increasing its speed for 1 sec. then slows down and stops 5 m left of 0 and passing through 0 between 2 and 3 seconds. It then increases its speed moving right of 0 indefinitely.

3 (a)(i) $P_1 = 2000e^{0.138t}$ $\quad P_2 = 5000e^{0.04t}$

when $P_1 = 4000$

$4000 = 2000e^{0.138t}$

$\therefore \ln 2 = 0.138t$

$\therefore t = 5.023$

$\therefore t = 5$ years (nearest year)

(ii) When $P_1 = P_2$

$2000e^{0.138t} = 5000e^{0.04t}$

$\therefore e^{0.098t} = \frac{5}{2}$

$\therefore 0.098t = \ln 2.5$

$\therefore t = 9.349$

$= 9$ years 5 months

$\therefore$ During May 2015

(iii) $\frac{dP_1}{dt} = 2000 \times 0.138 e^{0.138t}$

$= 276e^{0.138t}$

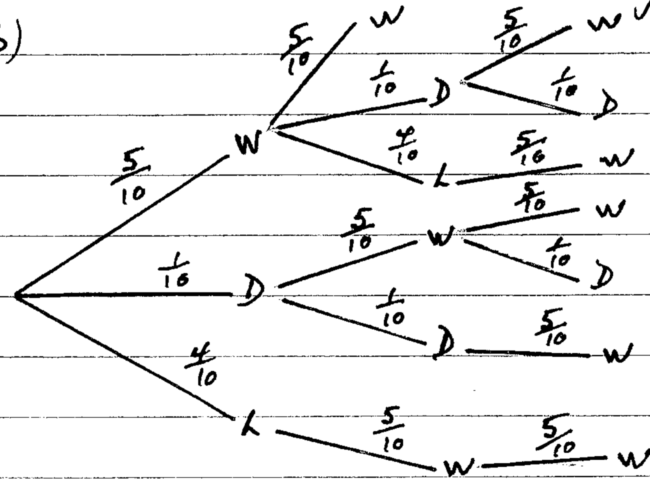
When $t = 9.349$

$\frac{dP_1}{dt} = 276e^{1.290162}$

$= 1002.8$

≈ 1003 / year.

(b)



$P = \left(\frac{5}{10} \times \frac{5}{10}\right) + \left(\frac{5}{10} \times \frac{1}{10} \times \frac{5}{10}\right) + \left(\frac{5}{10} \times \frac{1}{10} \times \frac{1}{10}\right) + \left(\frac{5}{10} \times \frac{4}{10} \times \frac{5}{10}\right)$

$+ \left(\frac{1}{10} \times \frac{5}{10} \times \frac{5}{10}\right) + \left(\frac{1}{10} \times \frac{5}{10} \times \frac{1}{10}\right) + \left(\frac{1}{10} \times \frac{1}{10} \times \frac{5}{10}\right) + \left(\frac{4}{10} \times \frac{5}{10} \times \frac{5}{10}\right)$

$= \frac{25}{100} + \frac{25}{1000} + \frac{5}{1000} + \frac{100}{1000} + \frac{135}{1000}$

$\therefore P = \frac{103}{200}$

3 (c) (i) $v = \frac{dx}{dt} = -e^{-2t}$

$\therefore x = -\int e^{-2t}\,dt$

$\therefore x = \frac{1}{2}e^{-2t} + c$

When $t=0$, $x=0$

$\therefore 0 = \frac{1}{2} + c \quad \therefore c = -\frac{1}{2}$

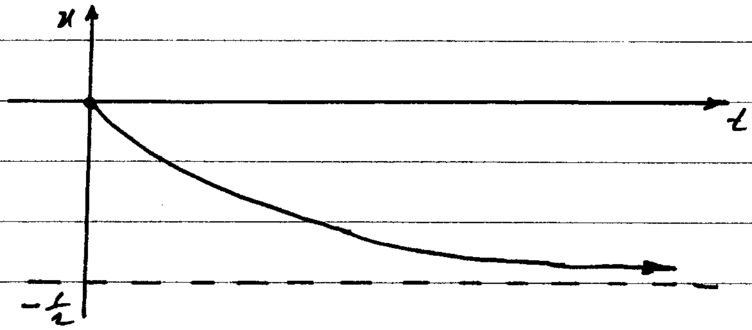
$\therefore x = \frac{1}{2}e^{-2t} - \frac{1}{2}$

$\therefore x = \frac{1}{2}\left(e^{-2t} - 1\right)$

(ii) When $t=0$, $x=0$

Let $x = \frac{1}{2}\left(\frac{1}{e^{2t}} - 1\right)$

As $t \to \infty$, $\frac{1}{e^{2t}} \to 0 \quad \therefore t \to -\frac{1}{2}$



4 (a) $\frac{x^2 - x + 1}{x^2 + x + 1} = t$

$\therefore x^2 - x + 1 = tx^2 + tx + t$

$\therefore (t-1)x^2 + (t+1)x + (t-1) = 0$

For real and different roots, we require $\Delta > 0$

Now $\Delta = (t+1)^2 - 4(t-1)^2$

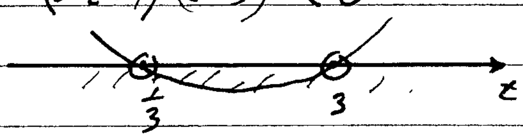
$= t^2 + 2t + 1 - 4t^2 + 8t - 4$

$= -3t^2 + 10t - 3$

$= -3(t^2 - 10t + 3)$

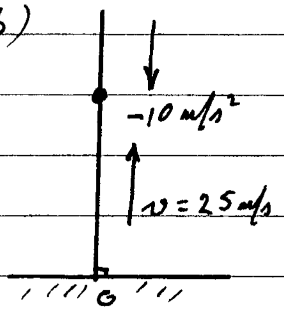
For $\Delta > 0$

$(3t-1)(t-3) < 0$



$\therefore \frac{1}{3} < t < 3$

(b)



$v = \frac{dx}{dt} = 25$

$\therefore x = \int 25\,dt$

$\therefore x = 25t + c$

When $t=0$, $x=0 \quad \therefore c=0$

$\therefore x = 25t$

Now $\frac{dv}{dt} = -10$

$\therefore v = -\int 10\,dt$

$\therefore v = -10t + c$

When $t=0$, $v=0 \quad \therefore c=0$

$\therefore v = -10t$

(i) Now $\underline{v = 25 - 10t}$ (since $v\uparrow = 25\ m/s$)

$\therefore \frac{dx}{dt} = 25 - 10t$

$\therefore x = \int (25 - 10t)\,dt$

$\therefore x = 25t - 5t^2 + c$

When $t=0$, $x=0 \quad \therefore c=0$

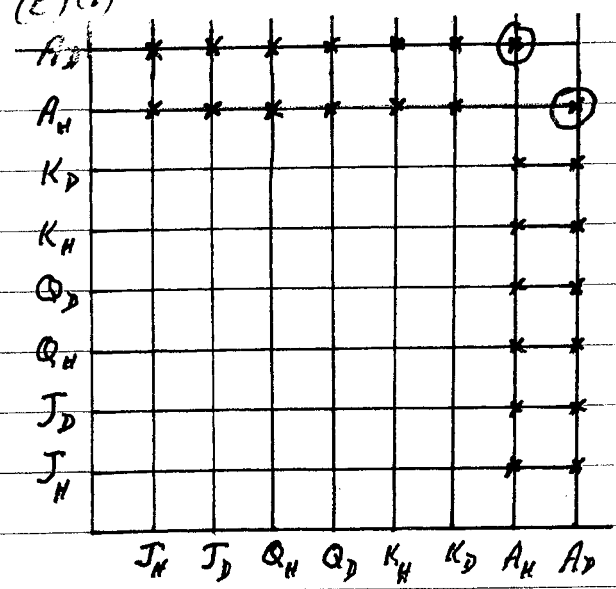
$\therefore \underline{x = 25t - 5t^2}$

(ii) Maximum height when $v=0$, ie, when $t = \frac{5}{2}$

When $t = \frac{5}{2}$, $x = 25\left(\frac{5}{2}\right) - 5\left(\frac{5}{2}\right)^2$

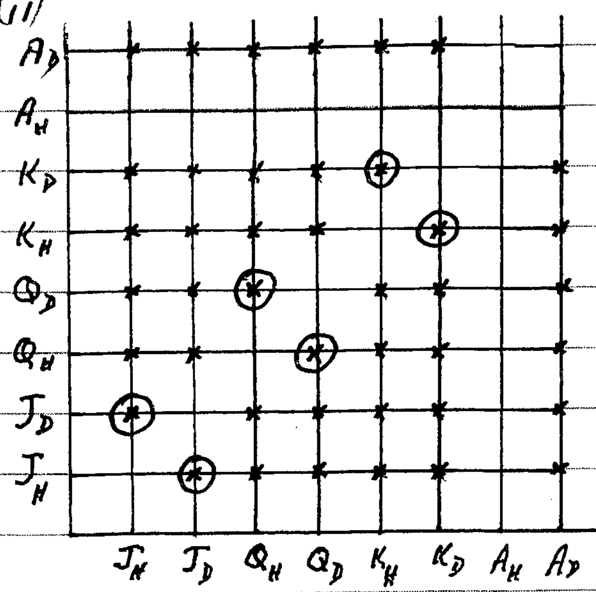
$\therefore x = 31\frac{1}{4}$ m.

4 (c) (i)



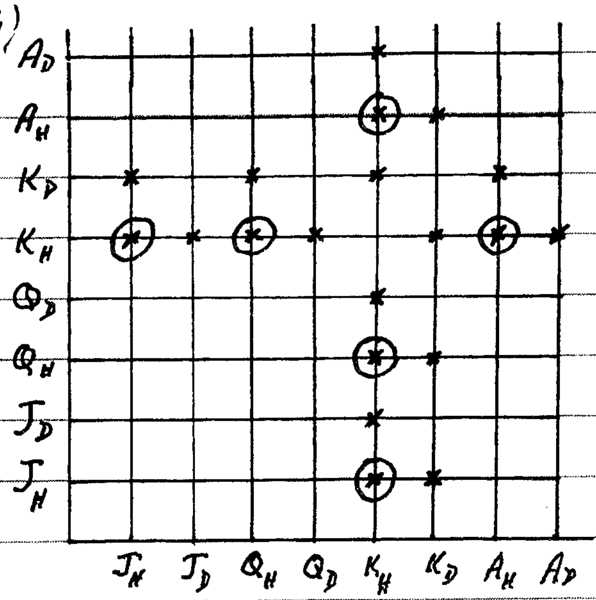
$$P = \frac{2}{26} = \frac{1}{13}$$

(ii)



$$P = \frac{6}{42} = \frac{1}{7}$$

(iii)



$$P = \frac{6}{20} = \frac{3}{10}$$