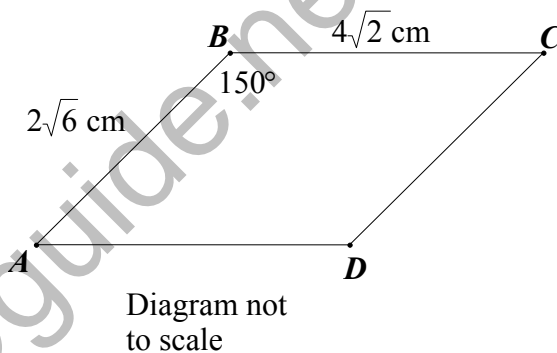


Year 12 Trial HSC Examination - Mathematics (2U) 2006

Question 1

- | | |
|--|------------|
| (a) Find the value of $\log_6 12$ correct to two decimal places. | Marks
2 |
| (b) The investment value of a stamp collection was originally \$24 800. If the value of the collection compounds at a rate of 5.2% annually, find its estimated value at the end of 10 years. Give your answer correct to the nearest \$100. | 2 |
| (c) The volume V of a cylinder with a base radius r and height h is given by the formula $V = \pi r^2 h$. Find the height of a cylinder when the volume is 480 cm^3 and base radius 9.5 cm. Give your answer correct to the nearest millimetre. | 2 |
| (d) Solve the equation $3(2x + 1) - 2(3 - x) = 53$. | 3 |
| (e) Find the exact length of the longer diagonal in parallelogram $ABCD$. | 3 |



Question 2 (Start a new page)

The coordinates of the points E , F and G are $(0,3)$, $(6,0)$ and $(7,-4)$ respectively.

- | | |
|--|------------|
| (a) Draw a neat sketch, clearly showing the above information and show that the line k which is parallel to EF and passes through point G , has the equation $x + 2y + 1 = 0$. | Marks
3 |
| (b) Find the coordinates of the point H where line k meets the x -axis. Clearly mark point H and the position line k on your diagram. | 2 |
| (c) Find the exact length of EF . | 2 |
| (d) Find perpendicular distance from point F to line GH . | 2 |
| (e) Find the exact area of quadrilateral $EFGH$. | 3 |

Question 3 (Start a new page)

	Marks
(a) Differentiate with respect to x :	
(i) $\frac{4x^3\sqrt{x} + 5}{x^2}$,	2
(ii) $(8 - 3x)^5$.	2
(b) Find the equation of the tangent to $y = xe^{-x}$ at the point where $x = 2$. Write your answer in general form.	4
(c) Differentiate $y = \frac{\cos x}{1 + \sin x}$,	4
and hence show that $\frac{dy}{dx} = \frac{-1}{1 + \sin x}$.	

Question 4 (Start a new page)

	Marks
(a) Evaluate	
(i) $\int_0^4 \frac{dx}{3x + 2}$,	2
(ii) $\int_{-1}^2 \cos\left(\frac{\pi}{2}x\right) dx$.	2
(b) (i) Find the coordinates of the stationary points on the curve $y = (x + 1)(x^2 - 8)$ and determine their nature.	5
(ii) Draw a neat half page sketch of $y = (x + 1)(x^2 - 8)$ in the domain $-4 \leq x \leq 4$. On your diagram clearly indicate the coordinates and the positions of the intercepts with the coordinate axes and the stationary points.	3

Question 5 (Start a new page)

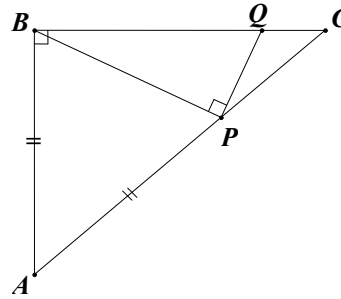
	Marks
(a) Town A is 4 km from town P and its bearing from town P is $030^\circ T$. Town B is due south of town A and 6 km from town P .	
(i) Draw a neat sketch that clearly illustrates the above information.	2
(ii) Find the distance between towns A and B . Give your answer correct to the nearest kilometre.	4
(b) (i) Find the intersection points of the line $y = 2x$ and the parabola $y = 6x - x^2$.	2
(ii) Find the area bounded by the line $y = 2x$ and the parabola $y = 6x - x^2$.	4

Question 6 (Start a new page)

	Marks
(a) A particle is moving along a straight line. The particle is initially at the origin O and at time t minutes its velocity, v m/min, is given by $v = 1 + 2 \sin\left(\frac{\pi}{4}t\right)$.	
(i) Sketch the graph of v against t for $0 \leq t \leq 8$.	2
(ii) Find the position of the particle when it first reaches its maximum speed.	4
(b) The gradient function of a curve is given by $\frac{dy}{dx} = 3 + \frac{10}{\sqrt{x}}$. Find the equation of the curve if it passes through the point $P(4,9)$.	3
(c) (i) Write down the discriminant (Δ) of the quadratic equation $x^2 + mx + m = 0$.	1
(ii) Hence, or otherwise, find the values of k for which the line $y = x + k$ and the curve $y = \frac{x}{x+1}$ do not intersect.	2

Question 7 (Start a new page)

- (a) $\triangle ABC$ and $\triangle BPQ$ are right-angled triangles and $AB = AP$.



- (i) Copy the diagram onto your answer sheet and prove that $\angle CBP = \angle CPQ$.
(Hint: Let $\angle CBP = \alpha^\circ$)
- (ii) Hence prove that $PC^2 = BC \times QC$
- (b) A mass of wire cable is wound around a winch. The mass, M kg, of wire on the winch, at time t minutes, is given by the formula $M = 400 - 25\sqrt{t + 100}$.
- (i) Find the initial mass of wire on the winch.
- (ii) Find the time needed to remove all the wire from the winch.
- (iii) Find the initial rate at which the wire is being removed from the winch.

Marks

3

3

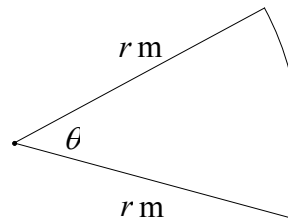
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2

2

Question 8 (Start a new page)

- (a) John starts work on an annual salary of \$38 500. On the last day of each year for the first 10 years of employment, he will receive an annual salary increase of \$1 200. For his remaining years of employment he will receive an annual increase of \$1 650.
- (i) What will John's annual salary be during his 6th year of employment?
- (ii) What will John's annual salary be during his 15th year of employment?
- (iii) How much will John's total earnings amount to at the end of his 25th year of employment?
- (b) A wire framework enclosing an area of 144 m^2 is to be made in the shape of a sector of a circle (see diagram). The length of wire required is L metres where the circle has radius r metres and the sector angle is θ radians.



- (i) Show that $L = 2r + \frac{288}{r}$.
- (ii) Find the minimum length of wire needed to make the framework.

Marks

2

2

2

2

4

Question 9 (Start a new page)

	Marks
(a) Point T is 6 cm from the centre O of a circle with radius 3 cm. Tangents drawn from point T touch the circle at points M and N and the tangents are perpendicular to the radii at these points of contact.	
(i) Find the exact size of $\angle MOT$.	2
(ii) Find the exact area of the minor sector MON .	3
(b) (i) Sketch the curve $y = 2 + \sqrt{x}$ and clearly shade the area bounded by the curve, the coordinate axes and the line $x = 9$.	2
(ii) Find the exact volume of the solid formed when the above area is rotated one revolution about the y-axis .	5

Question 10 (Start a new page)

	Marks
(a) In order to enter a university each student is required to sit for a theory examination. If a student passes the theory examination they are then required to sit for a practical examination. It is estimated that the probability for a student to pass the theory examination is 0.8 and to pass the practical examination is 0.75.	
(i) Find the probability that a student chosen at random will gain entry to the university.	2
(ii) Find the probability that if two students chosen at random only one will gain entry to the university.	2
(b) At 6am a population was observed to be undergoing exponential growth and after time t hours the number of individuals N_G was given by $N_G = 100e^{0.3t}$. When the population reached 500, a virus attacked the population and from that time the population underwent exponential decay so that t hours after the entry of the virus, the number of individuals N_D present was given by $N_D = 500e^{kt}$ for some constant $k < 0$. When the population fell to its original value it was decreasing at a rate of 15 individuals per hour.	
(i) At what time of the day did the population reach 500? Give your answer correct to the nearest minute.	3
(ii) Find the value of the constant k .	2
(iii) At what time of the day did the population return to its original value?	3

This is the END of the examination paper