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JRAHS MATHEMATICS 2005

SOLUTIONS (2UNIT)
TRAIL

MARKS

Q1 (a) $\dfrac{8\pi}{(2+\sqrt[3]{2})}$ = 7.7046..

$= \underline{7.71}$ (3SF) — 1

①

b) $\dfrac{d}{dx}[5x + \tan x]$

$= \underline{5} + \underline{\sec^2 x}$ ② — 1, 1

(c) $P(E = \text{prime}) = \dfrac{8}{21}$ ② — 1, 1

$\{2, 3, 5, 7, 11, 13, 17, 19\}$

(d) $\int \sec 3x \tan x \, dx$ ②

$= \underline{\frac{1}{3}\sec 3x} \; \underline{+c}$ — 1, 1

(e) $\sec\frac{\pi}{6} = \sec 30°$ ①

$\frac{1}{\sqrt{3}/2}$ $= \underline{\dfrac{2}{\sqrt{3}}}$ — 1

f) $a = \dfrac{3+11}{2} = \underline{7}$ ② — 1

$b = 7 - 3 = \underline{4}$ — 1

g) $(3-\sqrt{5})^2 = 9 - 6\sqrt{5} + 5$

$= 14 - 6\sqrt{5}$ — 1

$\therefore -6\sqrt{5} = \sqrt{p}$

$p = 36 \times 5 = \underline{180}$ — 1

②

Q2 (a) (i) $y = \dfrac{\sin x}{x+1}$ ②

$\therefore \dfrac{dy}{dx} = \dfrac{\cos x.(x+1) - 1.\sin x}{(x+1)^2}$ — 1, 1

$= \dfrac{(x+1)\cos x - \sin x}{(x+1)^2}$

(ii) $y = \sqrt{1+e^{6x}} = (1+e^{6x})^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \frac{1}{2}(1+e^{6x})^{-\frac{1}{2}} \times 6e^{6x}$ — 1, 1

$= \dfrac{3e^{6x}}{\sqrt{1+e^{6x}}}$ ②

(iii) $y = x^3 \ln x$ ②

$\dfrac{dy}{dx} = 3x^2.\ln x + x^3.\frac{1}{x}$ — 1

$= 3x^2 \ln x + x^2$ — 1

(b) for D_f $3 - x > 0$ ①

$\therefore D_f = \{x : \underline{x < 3}\}$ — 1

(c) $\tan\theta = 0.3$

$\theta = 0.2914...$ or $\pi + 0.2914..$

$\therefore \theta = \underline{0.29}$ or $\underline{3.43}$ (2dp) — 1, 1

②

(d) $\int \frac{1}{3x} dx = \frac{1}{3}\ln x \, (+c)$ — 1

or $= \frac{1}{3}\ln(3x)(+c)$

(e) $\int_0^2 e^{-x} + 1 \, dx$

$= -e^{-x} + x\Big]_0^2$ — 1

$= (-e^{-2} + 2) - (-e^0 + 0)$

$= -e^{-2} + 2 + 1$ — 1

$= 3 - e^{-2}$

2.86466...

Q3 (a) $y = 2\cos x + 3$

$\dfrac{dy}{dx} = -2\sin x$

at $x = \frac{\pi}{2}$ $m_T = -2\sin\frac{\pi}{2} = -2$

$\therefore m_N = +\frac{1}{2}$ — 1

Eqn. of normal $(\frac{\pi}{2}, 3)$

$y - 3 = \frac{1}{2}(x - \frac{\pi}{2})$ — 1

$y = \frac{1}{2}x + 3 - \frac{\pi}{4}$ / $2x - 4y + 12 - \pi = 0$

②

b) (i) $m_{BD} = \dfrac{2-0}{0-8} = \underline{-\frac{1}{4}}$ — 1

①

(ii) Eqn. of BD : $y = -\frac{1}{4}x \underline{+ 2}$ — 1

$B(0,2)$ $[x + 4y - 8 = 0]$

①

iii) $m_{BD} = \tan\theta = -\frac{1}{4}$

$\therefore \angle\theta = 165°58'$

nearest degree $= \underline{166°}$ — 1

①

(iv) $m_{CP} = m_{BD} = -\frac{1}{4}$ ② — 1

Eqn. of CP : $y - 5 = -\frac{1}{4}(x-3)$

$4y - 20 = -x + 3$ — 1

$\therefore x + 4y - 23 = 0$

(v) at P $y = 0$

$\therefore x + 0 - 23 = 0 \Rightarrow x = 23$

$\therefore P = \underline{(23, 0)}$. — 1

vi) ⊥ dist. $= \dfrac{|1\times3 + 4\times5 - 8|}{\sqrt{1^2+4^2}}$ — 1

$C = (3,5)$

$x + 4y - 8 = 0$ ②

$= \underline{\dfrac{15}{\sqrt{17}}}$ units — 1

vii) Now $|ABCD| = |ABD| + |BDC|$

but $|BDP| = |BDC|$ (same base BD, same perp. height between parallel lines)

1 1

$\therefore |ABCD| = |ABD| + |BDP|$ ②

$= |ABP|$ q.e.d.

OR $|APB| = \frac{1}{2}\times24\times2 = 24\,u^2$

$|ABD| = \frac{1}{2}\times9\times2 = 9\,u^2$ (common)

Q4 (a) $\dfrac{p}{\sin 72°} = \dfrac{r}{\sin 69°}$ ②

so $\dfrac{p}{r} = \dfrac{\sin 72°}{\sin 69°} = 1.0187194\text{)}6..$

$= \underline{1.019}$ (3dp) — 1

(b) $x^2 - 2x - 5 = 0$ ②

$\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta} = \dfrac{+2}{-5} = -\dfrac{2}{5}$ — 1, 1

(c) (i) $y = x - 5$

at A $y = 0$ $\therefore \underline{x = 5}$ — 1

(ii)

Area $= \int_0^5 (Y_U - Y_L)\,dx$

$= \int_0^5 6x - x^2 - 5 - (x-5)\,dx$ — 1

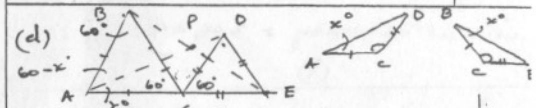
$= \int_0^5 (5x - x^2)\,dx$

$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$ ②

$= \frac{125}{2} - \frac{125}{3} - 0$

Area $= \frac{125}{6} = \underline{20\frac{5}{6}}\;u^2$. — 1

(d)



(i) In Δs ACD and ECD

S 1. AC = CB (all sides equal in equil. ΔABC) — 1

A 2. ∠ACD = ∠ECB = $\underline{\angle BCD + 60°}$ — 1

(all angles are 60° in equilateral Δs)

S 3. CD = CE ③

∴ Δ ACD ≡ Δ ECD ($\underline{SAS}$) — 1

(ii) $\underline{\text{Let}}$ ∠CAD = x°

1. ∠CBE = x° (corresp. angles in cong. triangles are equal)

2. ∠BAP = 60° − x°

3. ∠APB + 60° − x° + 60° + x° = 180° (Angles sum of Δ APB is 180°) — 1

② ∴ ∠APB = $\underline{60°}$

* $BD = \sqrt{68}$

and $|BDC| = \frac{1}{2}\times\sqrt{68}\times\dfrac{15}{\sqrt{17}} = 15\,u^2$

$|BPP| = \frac{1}{2}\times\sqrt{68}\times\dfrac{15}{\sqrt{17}} = 15$

$\therefore |ABCD| = 9 + 15$

$= 24$

$\therefore |ABCD| = |ABP| = 24\,u^2$

6

) $\log_3(2x-5) = 1$ ②

$\therefore \quad 2x - 5 = 3^1 = 3$ 1

$\therefore \underline{x = 4}$ 1

$f'(x) = \lim_{h\to 0} \dfrac{\left(\frac{3}{x+h} - \frac{3}{x}\right)}{h}$

$= \lim_{h\to 0}\left(\dfrac{3x - 3(x+h)}{hx(x+h)}\right)$ 1 ②

$= \lim_{h\to 0} \dfrac{-3\cancel{h}}{\cancel{h}x(x+h)}$ $h \neq 0$ as $h \to 0$

$= \lim_{h\to 0} \dfrac{-3}{x(x+h)}$ 1

$'(x) = \dfrac{-3}{x(x+0)} = -\dfrac{3}{x^2}$

$V = 3e^t + 6e^{-t}$ $\quad t=0,\ x=0$

initial velocity $= 3+6 = \underline{9\text{ m/s}}$ 1 ①

$e^t, e^{-t} > 0$ for all $t \geq 0$

$V = 3e^t + 6e^{-t} > 0$ ① 1

particle never at rest.
as $V \neq 0$

) $x = \int (3e^t + 6e^{-t})\,dt$

$x = 3e^t - 6e^{-t} + c$ 1

en $t = 0 \quad x = 0$

$\therefore 0 = 3 - 6 + c$ ②

$\therefore c = 3$ 1

so $x = 3e^t - 6e^{-t} + 3$

$\therefore x = 3e^t - 6e^{-t} + 3 = 10$

$3e^t - 6e^{-t} - 7 = 0$

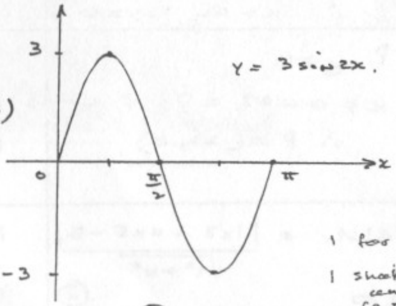
$3e^{2t} - 6 - 7e^t = 0$

$u = e^t$ 1

$3u^2 - 7u - 6 = 0$

$(3u+2)(u-3) = 0$

(d)



1 for Amp.

1 shape and $[0, \pi]$

②

$\therefore u = -\frac{2}{3}$ or $u = 3$

i.e. $e^t = -\frac{2}{3}$ or $e^t = 3$

but $e^t > 0$ or $\underline{t = \ln 3}$ 1

$\therefore$ no soln possible

$\therefore$ the time to be at $x = 10$

② is $\underline{\ln 3}$ seconds

Q5.

(a) $r = \cos^2 x \quad 0 < x < \frac{\pi}{2}$ $\quad 0 < r < 1$

$\therefore S = \dfrac{a}{1-r} = \dfrac{1}{1-\cos^2 x}$ ①

$S = \dfrac{1}{\sin^2 x} = \operatorname{cosec}^2 x$ 1

(b) (i) $T_n = L = a + (n-1)d$ ① 1

(ii) $S_n = \frac{n}{2}[2a + (n-1)d] \,//\, \frac{n}{2}[a + L]^*$

but as $L = a + (n-1)d$

$(n-1) = \dfrac{L-a}{d}$ 1

$n = \dfrac{L-a}{d} + 1$

so $S_n = \frac{1}{2}\left(\frac{L-a}{d} + 1\right)\left[2a + \left(\frac{L-a}{d}\right)\times d\right]$

$= \frac{1}{2}\left(1 + \frac{L-a}{d}\right)[2a + L - a]$ 1

$\therefore S_n = \frac{1}{2}(a+L)\left(1 + \frac{L-a}{d}\right)$ ②

(iii) Hence: ①

$S = \frac{1}{2}(5 + 173)\left(1 + \frac{173-5}{3}\right)$

$= \frac{1}{2} \times 178 \times (1 + 56)$

Sum $= \underline{5073}$. 1

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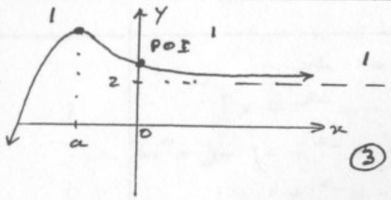
c)



③

1 max TP

1 POI* $x=0$

1 HA $y=2$

* as shape of $f(x)$

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d) 1. $\angle BAH = \frac{(6-2)}{6} \times 180° = 120°$

(all angles of reg. hexagon 120°) 1

2. $\angle BAP = \left(\frac{5-2}{5}\right) \times 180° = 108°$

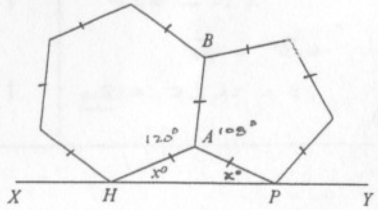
(all angles of reg. pentagon 108°)

3. $\angle HAP + 120° + 108° = 360°$

(angle sum at pt. A is 360°)

$\therefore \angle HAP = 132°$ 1

④



4. $AH = AP = AB$ (all sides of reg. hexagon + pentagon are equal) 1

5. $\angle APH = \angle AHP = x°$ (equal angles opposite equal sides)

6. $x + x + 132 = 180$ (angle sum of $\triangle AHP = 180°$)

$2x = 48$

$\underline{x = 24}$ 1

$OP = PM$

$\sqrt{x^2+y^2} = |4-y|$

$x^2+y^2 = (4-y)^2$ 1

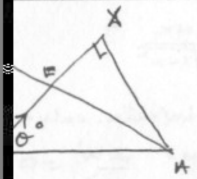
$x^2+y^2 = 16-8y+y^2$ 1

$x^2 = 16-8y$

$= 16-8y$

$= 4(-2)(y-2)$

$\underline{(0,2)}$ ① 1



$D = \angle XBC$

nates angles equal 1

$DC \parallel BX$ ①

$BX = \theta°$ (Data) 1

$DB = \theta°$ (corresponding angles equal as $DC \parallel BX$) 1

$\angle CDB = \angle DCB = \theta°$

$\triangle BCD$ is isosceles ②

angles equal)

$= \dfrac{AB}{BD}$ 1

$B = CB$ (equal sides opposite equal angles) 1

$\dfrac{E}{C} = \dfrac{AB}{CB}$ q.e.d.

②

METHOD 1.

(iv) let $BC = a = BD$

$\therefore BA = 3a$

and so $AE = 3EC$ part (iii) 1

$AE^2 = 9EC^2$

i.e $BE^2 + 9a^2 - 6aBE\cos\theta$

$= 9[a^2 + BE^2 - 2aBE.\cos\theta]$ 1

$BE^2 + \cancel{9a^2} - 6aBE\cos\theta = \cancel{9a^2} + 9BE^2 - 18aBE\cos\theta$

$\therefore 8BE^2 = 12aBE\cos\theta$

$2BE = 3a\cos\theta$ 1

but $\cos\theta = \dfrac{BX}{BA} = \dfrac{BX}{3a}$ ③

$\therefore 2BE = BX$

$\Rightarrow$ E is the midpt of BX

Q7.(a)

(i) $m_L = \dfrac{1-0}{0-\frac{\pi}{2}} = -\dfrac{2}{\pi}$ 1

(0,1)

$(\frac{\pi}{2}, 0)$

$\therefore$ Equ. of L : $y = -\frac{2}{\pi}x + 1$.

(ii) Volume $= \pi\int_0^{\pi/4} \sec^2 x\,dx - \pi\int_0^{\pi/4} (1-\frac{2}{\pi}x)^2\,dx$

$= \pi\int_0^{\pi/4} \sec^2 x - (1-\frac{2}{\pi}x)^2\,dx$

or $= \pi\int_0^{\pi/4} \sec^2 x - 1 + \frac{4}{\pi}x - \frac{4}{\pi^2}x^2\,dx$

$= \pi\left[\tan x + \frac{\pi}{6}(1-\frac{2}{\pi})^3\right]_0^{\pi/4}$ 1

or $= \pi\left[\tan x - x + \frac{2}{\pi}x^2 - \frac{4}{3\pi^2}x^3\right]_0^{\pi/4}$

$= \pi\left[(1+\frac{\pi}{6}(1-\frac{1}{2})^3) - (0+\frac{\pi}{6})\right]$ 1

$= \pi\left[1 + \frac{\pi}{48} - \frac{\pi}{6}\right]$ ③

$\underline{Vol} = \underline{\pi\left[1 - \frac{7\pi}{48}\right]}\ u^3$. 1

(b) $I = \int_0^4 \dfrac{3\,dx}{1+\sqrt{x}}$

x	0	2	4
$\frac{3}{1+\sqrt{x}}$	3	$\frac{3}{1+\sqrt{2}}$ 1.2426...	1

$\therefore I \doteqdot \frac{2}{3}\left[3 + 1 + 4\times\frac{3}{1+\sqrt{2}}\right]$ 1

$= \frac{2}{3} \times 8.97056...$

$= 5.980375...$ ②

$\therefore I = \underline{5.98}$ (2dp) 1

(c) (i) $f'(x) = 3x^2 - 3k$

If $k<0$ then $\underline{-3k>0}$, $3x^2 \geq 0$ 1

so $f'(x) = 3x^2 - 3k > 0$

$\therefore$ $f(x)$ is increasing $\forall x$. 1

②

(c) (ii)

For S.Ps to occur $f'(x) = 0$

$\therefore 3x^2 - 3k = 0$

$x^2 = k$

$k>0$ $x = \sqrt{k}$ or $-\sqrt{k}$

so $f(\sqrt{k}) = (\sqrt{k})^3 - 3k\sqrt{k} + 4$

$= k\sqrt{k} - 3k\sqrt{k} + 4$

$= 4 - 2k\sqrt{k}$

and $f(-\sqrt{k}) = (-\sqrt{k})^3 - 3k(-\sqrt{k}) + 4$

$= -k\sqrt{k} + 3k\sqrt{k} + 4$

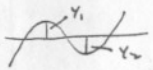
$= 4 + 2k\sqrt{k}$

$\therefore$ SPs are $(\sqrt{k}, 4-2k\sqrt{k})$ 1

② $(-\sqrt{k}, 4+2k\sqrt{k})$ 1

(iii) Since a cubic

For 3 distinct real roots



$Y_1 Y_2 < 0$ 1

so $(4-2k\sqrt{k})(4+2k\sqrt{k}) < 0$

$16 - 4k^2 \times k < 0$

$16 - 4k^3 < 0$

② $-4k^3 < -16$ 1

$\therefore \underline{k^3 > 4}$

(OR) as $k>0$

$4 + 2k\sqrt{k} > 0$

but $4 - 2k\sqrt{k} < 0$ for 3 roots

i.e $2k\sqrt{k} > 4$

$k\sqrt{k} > 2$

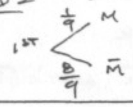
$\therefore \underline{k^3 > 4}$

10.

) (i) METHOD 1. Not match.

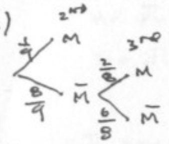
$P(E) = 10 \times \frac{1}{10} \times \frac{10-2}{10-1} = \frac{8}{9}$ 1

ETHOD 2



$P(E) = 8/9$ ①

i)



1

P(E = no match after 3 socks).

$= \frac{8}{9} \times \frac{6}{8} = \frac{2}{3}$ ② 1

i) P(E = a matching pair in 3)

$= 1 - P(\text{no match})$ 1

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$ ①

or $P(MM) + P(M\bar{M}M) + P(\bar{M}MM)$

$5 \times [\frac{1}{10} \times \frac{1}{9} + \frac{2}{10} \times \frac{8}{9} \times \frac{1}{9} + \frac{8}{10} \times \frac{2}{8} \times \frac{1}{9}] = \frac{1}{3}$

) (i) $r = 1\%$ p mnth ②

interval k gives $\frac{12}{k}$

∴ $n = 7k$

Amount owing at end of 1st \$F

$A_1 = 30\,000 \times 1.01^{12/k} - F$ 1

Amt. owing at end of 2nd \$F

$A_2 = A_1 \times 1.01^{12/k} - F$ 1

$= 30\,000 \times 1.01^{2 \times 12/k} - F(1 + 1.01^{12/k})$

$A_2 = 30\,000 \times [1.01^{\frac{12}{k}}]^2 - F[1 + 1.01^{12/k}]$

) Amt owing at end of n^{th} \$F

$A_n = 30\,000(1.01^{12/k})^n - F(1 + 1.01^{12/k} + 1.01^{24/k} + \dots + 1.01^{(n-1)\frac{12}{k}}]$

but when $\frac{12}{k} \times n = 84$ or $n = 7k$ 1

$A_{7k} = 0$ debt paid

Q 10 (b) (ii) ③

$0 = 30\,000 \times 1.01^{84} - F[1 + 1.01^{\frac{12}{k}} + \dots + 1.01^{(7k-1)12/k}]$

$0 = 30\,000 \times 1.01^{84} - F \times a\frac{(r^n - 1)}{r - 1}, n = 7k$ 1

i.e. $F \times [\frac{(1.01^{12/k})^n - 1}{1.01^{12/k} - 1}] = 30\,000 \times 1.01^{84}$ 1

i.e. $F \times [\frac{1.01^{84} - 1}{1.01^{12/k} - 1}] = 30\,000 \times 1.01^{84}$

∴ $F = \frac{30\,000 \times 1.01^{84}(1.01^{12/k} - 1)}{(1.01^{84} - 1)}$

(iii) $k = 4$ $F_4 = 1604.69$ 1

$\frac{12}{k} = 3$ ①

∴ 3 mths

(iv) $k = 12$, $\frac{12}{k} = 1$

$F_{12} = 529.58$ 1

Save $= 28F_4 - 84F_{12}$ 1

$= \$446.33$ ②

Q9.

(a) (i) $\frac{dM}{dt} = -kM$

LHS RHS

$\frac{d}{dt}M_0e^{-kt}$ $-kM = -kM_0e^{-kt}$ 1

$= -kM_0e^{-kt}$

∴ LHS = RHS

∴ $M = M_0e^{-kt}$ satisfies $\frac{dM}{dt} = -kM$.

(ii) $t = T$ $M = \frac{1}{2}M_0$

∴ $\frac{1}{2}M_0 = M_0e^{-kT}$

$\frac{1}{2} = e^{-kT}$ ②

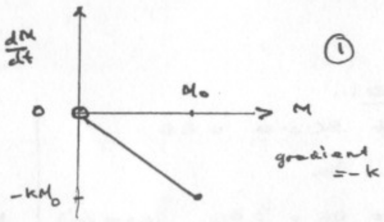
$\ln\frac{1}{2} = -kT$ 1

$T = \frac{\ln\frac{1}{2}}{-k} = -\frac{\ln 2}{-k} = \frac{\ln 2}{k}$ 1

(iii) $3466 = \frac{\ln 2}{k}$ ①

$k = \frac{\ln 2}{3466} = 2 \times 10^{-4}$ (1SF) 1

(iv) ①



1 negative gradient line segment

b) (i) $W = \frac{|3.x + 4.y + 24|}{\sqrt{3^2 + 4^2}}$ 1

$= \frac{|3x + 4y + 24|}{5}$

but as P on the RHS of the line $3x + 4y + 24 > 0$ 1

∴ $W = \frac{3x + 4y + 24}{5}$. qed. ②

Q9 (b) (ii) METHOD 1.

$P(x,y)$ lies on $x^2 + y^2 = 1$

∴ $y = \pm\sqrt{1 - x^2}$

but shortest distance will occur on lower semi circle 1

∴ $y = -\sqrt{1 - x^2}$

∴ $W = \frac{1}{5}(3x - 4\sqrt{1 - x^2} + 24)$

$\frac{dW}{dx} = \frac{1}{5}[3 - 4 \times \frac{1}{2}(1 - x^2)^{-\frac{1}{2}} \times -2x]$ 1

$= \frac{1}{5}[3 + \frac{4x}{\sqrt{1 - x^2}}]$

For possible max/min. values of W to occur $\frac{dW}{dx} = 0$

∴ $3 + \frac{4x}{\sqrt{1 - x^2}} = 0$

so $4x = -3\sqrt{1 - x^2}$

so $x < 0$ for solns

∴ $16x^2 = 9(1 - x^2)$

$16x^2 = 9 - 9x^2$

$25x^2 = 9$

$x^2 = \frac{9}{25}$

$x = \pm\frac{3}{5}$ 1

but $x < 0$ $x = -\frac{3}{5}$, $y = -\frac{4}{5}$ 1

TEST at $x = \frac{-3}{5}$

$\frac{3 + \frac{4x}{\sqrt{}}}{5}$

x	-0.7	$-\frac{3}{5}$	$-\frac{1}{2}$
W'	-0.184	0	0.138
	\ ⊖	—	⊕ /

$-0.7 \le x \le -\frac{1}{2}$

Since W is cont + diff'ble over and $\frac{dW}{dx}$ changes sign $(-\,0\,+)$

∴ a relative min. T.P at $x = -\frac{3}{5}$

since no other TPs in $-1 \le x \le 0$

∴ (abs) min. length is

⑤ $W = \frac{1}{5}[3(-\frac{3}{5}) + 4(-\frac{4}{5}) + 24]$

$= 3.8$ units 1