



2010 Trial Examination

FORM VI

MATHEMATICS 2 UNIT

Tuesday 3rd August 2010

General Instructions

- Reading time — 5 minutes
- Writing time — 3 Hours
- Write using black or blue pen.
- Board-approved calculators may be used.
- A list of standard integrals is provided at the end of the examination paper.
- All necessary working should be shown in every question.
- Start each question in a new booklet.

Structure of the paper

- Total marks — 120
- All ten questions may be attempted.
- All ten questions are of equal value.

Collection

- Write your candidate number clearly on each booklet.
- Hand in the ten questions in a single well-ordered pile.
- Hand in a booklet for each question, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place the question paper inside your answer booklet for Question One.

Checklist

- SGS booklets — 10 per boy
- Candidature — 85 boys

Examiner
SO

QUESTION ONE (12 marks) Use a separate writing booklet.

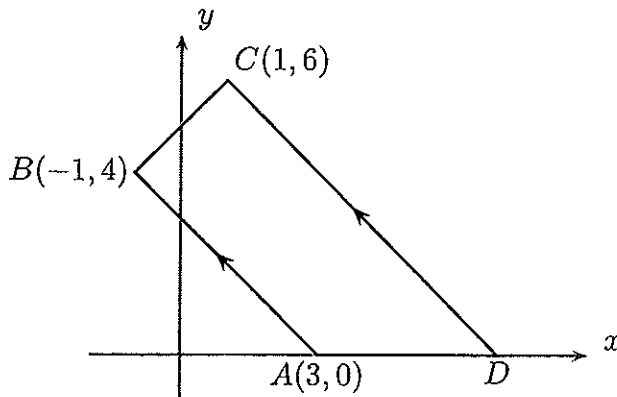
Marks

- (a) Find the value of $\frac{3.6 \times 7.4}{\sqrt{5.6 + 2.5}}$ correct to 2 significant figures. **1**
- (b) Factorise $x^3 - 125$. **1**
- (c) If $(\sqrt{7} - 3)(2\sqrt{7} + 2) = p + q\sqrt{7}$, find p and q . **2**
- (d) Simplify $\frac{x}{3} - \frac{x+2}{4}$. **2**
- (e) Solve $|x - 1| < 3$. **2**
- (f) Solve $\cos \theta = \frac{\sqrt{3}}{2}$, for $0 \leq \theta \leq 2\pi$. **2**
- (g) Find the sum of the first 17 terms of the arithmetic series $3 + 11 + 19 + \dots$. **2**

QUESTION TWO (12 marks) Use a separate writing booklet.

Marks

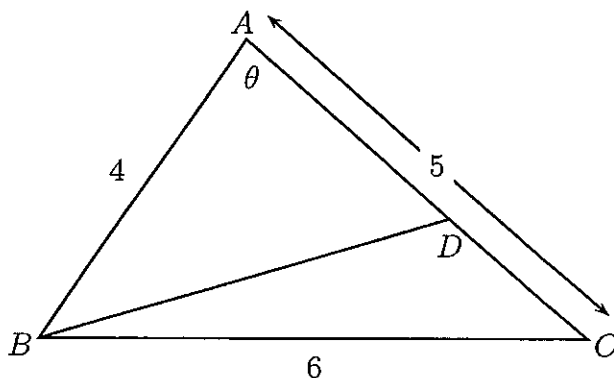
(a)



In the diagram above, $ABCD$ is a trapezium with $AB \parallel DC$. The coordinates of A , B and C are $(3, 0)$, $(-1, 4)$ and $(1, 6)$ respectively. D lies on the x -axis.

- (i) Find the length of AB . 1
- (ii) Find the gradient of AB . 1
- (iii) Find the equation of the line CD , and hence find the coordinates of D . 2
- (iv) Show that the perpendicular distance from A to the line CD is $2\sqrt{2}$ units. 2
- (v) Hence, or otherwise, calculate the area of the trapezium $ABCD$. 2

(b)



In the diagram above $\angle BAC = \theta$ as shown.

- (i) Find the exact value of $\cos \theta$. 2
- (ii) The point D lies on AC . Given that $AD = 3$, calculate the exact length of BD . 2

QUESTION THREE (12 marks) Use a separate writing booklet.

Marks

(a) Differentiate the following functions:

(i) $y = 3x^2 + \frac{1}{x}$ 2

(ii) $y = 3(2x - 5)^4$ 2

(iii) $y = x \tan x$ 2

(b) Find the equation of the tangent to the curve $y = \log_e x$ at $(e, 1)$. 2

(c) Find $\int \sec^2 \frac{1}{3}x dx$. 2

(d) Evaluate $\int_0^1 \frac{4}{4x + 1} dx$. 2

QUESTION FOUR (12 marks) Use a separate writing booklet.

Marks

(a) Consider the parabola $x^2 = 4(y - 2)$.

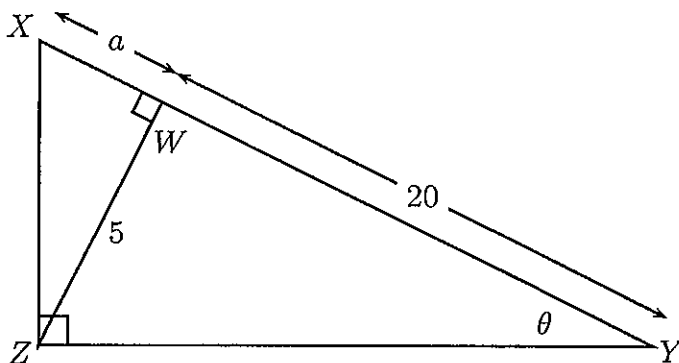
(i) Write down the coordinates of the vertex.

1

(ii) Find the coordinates of the focus.

1

(b)



The diagram above shows $\triangle XYZ$ which is right-angled at Z . The interval ZW is perpendicular to XY . Let $\angle WYZ = \theta$.

(i) Show that $\angle WZX = \theta$.

1

(ii) Hence prove that $\triangle WZX$ is similar to $\triangle WYZ$.

2

(iii) Let $XW = a$. If $WZ = 5$ and $WY = 20$, find a .

1

(c) Let α and β be the roots of $3x^2 - 4x - 2 = 0$.

(i) State the value of $\alpha\beta$.

1

(ii) Find $\frac{5}{\alpha} + \frac{5}{\beta}$.

2

(d) The second term of a geometric series is 270 and the fifth term is 80.

(i) Find the common ratio and the first term of the series.

2

(ii) Find the limiting sum of the series.

1

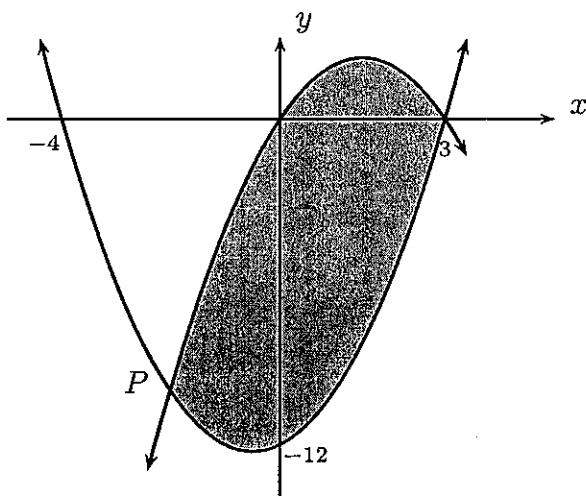
QUESTION FIVE (12 marks) Use a separate writing booklet.

Marks

(a) Consider the curve $y = x^3 - 3x + 2$.

- (i) Find the coordinates of the stationary points and determine their nature. 3
- (ii) Find any points of inflexion. 2
- (iii) Sketch the curve, showing the stationary points and any points of inflexion. 2
- (iv) For what values of x is the curve concave down? 1

(b)



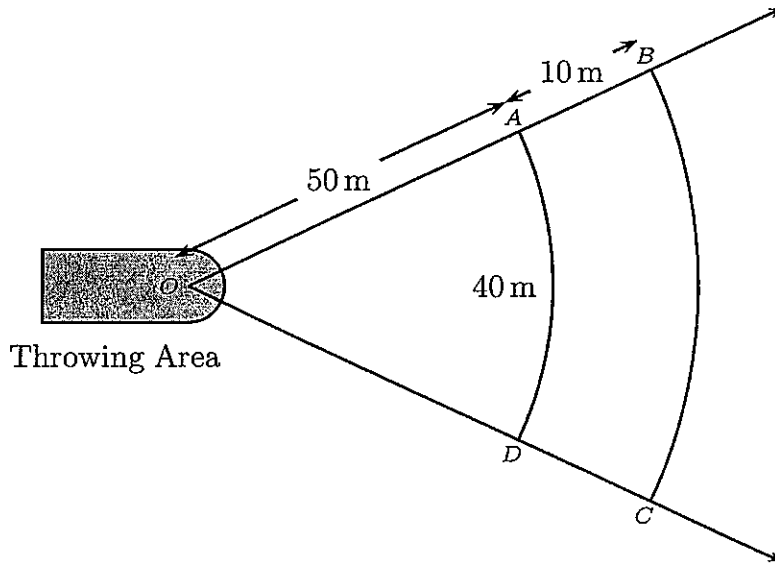
The graphs of the functions $y = x^2 + x - 12$ and $y = -x^2 + 3x$ are shown in the diagram above. They intersect at $(3, 0)$ and at P .

- (i) By solving simultaneously, show that P has x -coordinate -2 . 1
- (ii) Calculate the area of the shaded region. 3

QUESTION SIX (12 marks) Use a separate writing booklet.

Marks

(a)



The diagram above shows the javelin competition area at an athletic stadium. The circular arcs AD and BC have centre O . The arc AD has length 40 metres and radius 50 metres.

(i) Calculate the size of $\angle AOD$ in radians.

2

(ii) The worst throw of the day landed on the arc AD and the best throw of the day landed on the arc BC . If $AB = 10$ metres, calculate the area of the region $ABCD$ in which all the other throws landed.

2

(b) Julian's house is being overrun with Black European cockroaches. Assume that without intervention the population P of cockroaches grows exponentially according to the equation $P = Ae^{kt}$, where A and k are constants, and t is the time in days. When Julian leaves for a holiday there are 50 cockroaches in his house. After ten days the cockroach population has increased to 275.

(i) Show that $P = Ae^{kt}$ satisfies $\frac{dP}{dt} = kP$.

1

(ii) Find the exact value of k .

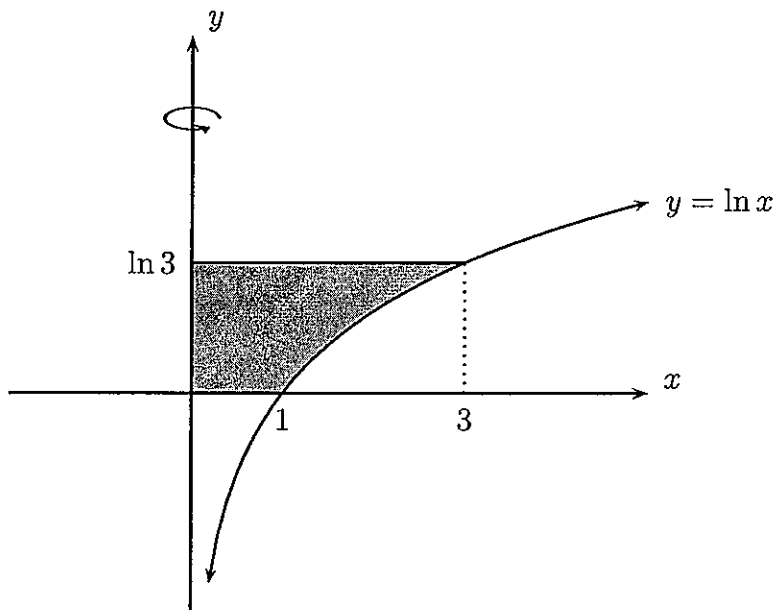
2

(iii) When the cockroach population exceeds 2000, the house will be declared an area of infestation. Julian returns from his holiday after 3 weeks. Will he discover an infestation when he arrives home?

2

(c)

3



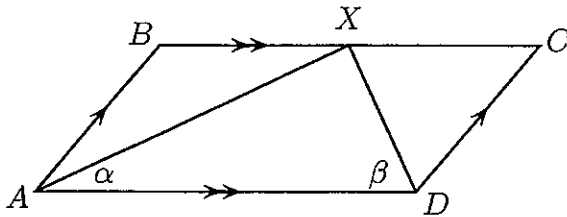
In the diagram above the shaded region is bounded by the curve $y = \ln x$, the x -axis, the y -axis and the line $y = \ln 3$. Calculate the exact volume of the solid formed when the shaded region is rotated about the y -axis.

QUESTION SEVEN (12 marks) Use a separate writing booklet.

Marks

(a) Find the value of m such that $\int_{\frac{1}{2}}^m \frac{1}{x^2} dx = 1$. 2

(b)



In the diagram above $ABCD$ is a parallelogram and X is on BC such that AX bisects $\angle BAD$ and DX bisects $\angle CDA$. Let $\angle XAD = \alpha$ and $\angle XDA = \beta$.

(i) Prove that $\triangle ABX$ is isosceles. 2

(ii) Prove that $\angle AXD = 90^\circ$. 2

(c) (i) Copy and complete the table for the function $y = x \sin x$, writing the y -values correct to four decimal places. 1

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y					

(ii) Use Simpson's rule with five function values to approximate $\int_0^\pi x \sin x dx$. Round your answer to two decimal places. 2

(iii) Show $\frac{d}{dx}(\sin x - x \cos x) = x \sin x$, and hence find the exact value of $\int_0^\pi x \sin x dx$. 2

(iv) Hence determine the percentage error in your approximation in part (ii). Write your answer correct to one decimal place. 1

QUESTION EIGHT (12 marks) Use a separate writing booklet.

Marks

(a) Given that $x = \frac{3}{4}$ is one root of the quadratic equation $mx^2 + 7x - m = 0$, find the other root. 2

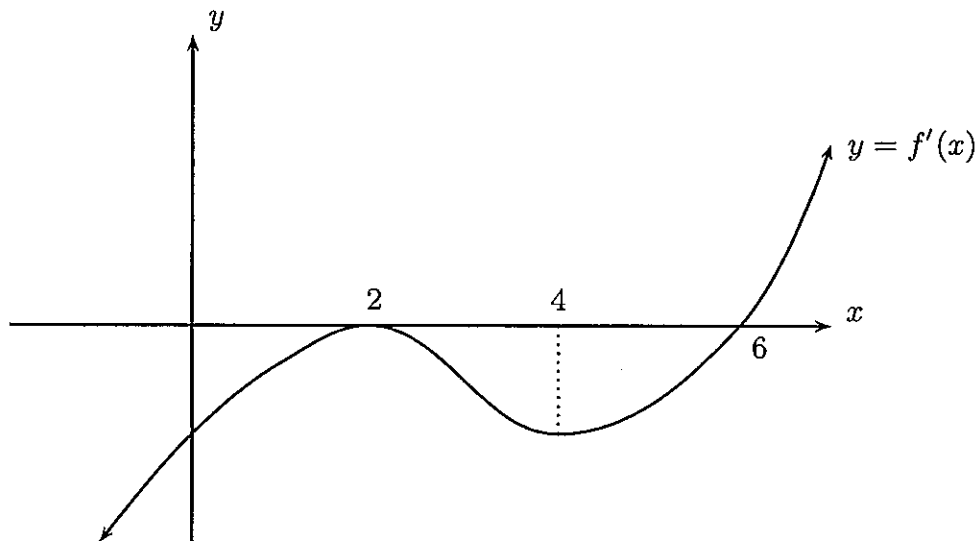
(b) Sophie has a toy that she uses to blow spherical bubbles. The rate of change of the volume $V \text{ cm}^3$ of a bubble is given by

$$\frac{dV}{dt} = \frac{6t}{t^2 + 1} \text{ cm}^3/\text{s}.$$

(i) Find the equation for the volume V of a bubble t seconds after Sophie starts blowing. Assume that the initial volume of a bubble is zero. 2

(ii) A bubble will burst when its radius exceeds 1.5 cm. Sophie takes a deep breath and blows a bubble. After how many seconds of blowing will it burst? Give your answer correct to one decimal place. 3

(c)



The diagram above shows the graph of the gradient function $y = f'(x)$ of the function $y = f(x)$.

(i) For what values of x is the function $y = f(x)$ increasing? 1

(ii) For what values of x is the curve $y = f(x)$ concave down? 1

(iii) Given that $f(0) = 2$, draw a possible sketch of $y = f(x)$. 3

QUESTION NINE (12 marks) Use a separate writing booklet.

Marks

- (a) Solve $2 \sin^2 \alpha - \cos \alpha + 1 = 0$, for $0 \leq \alpha \leq 2\pi$. 3

- (b) Solve $\log_6(x + 3) + \log_6(x - 2) = 2$. 3

- (c) A particle is moving along the x -axis . Its position at time t is given by $x = 5e^{-t} \sin t$.
 - (i) Show that its velocity is given by $v = 5e^{-t}(\cos t - \sin t)$. 1
 - (ii) Where is the particle initially, and what is its initial velocity? 2
 - (iii) At what time during the interval $0 \leq t \leq \pi$ is the particle stationary? 2
 - (iv) Assuming that its acceleration at time t is $\ddot{x} = -10e^{-t} \cos t$, find the time during the interval $0 \leq t \leq \pi$ when the acceleration is zero. 1

QUESTION TEN (12 marks) Use a separate writing booklet.

Marks

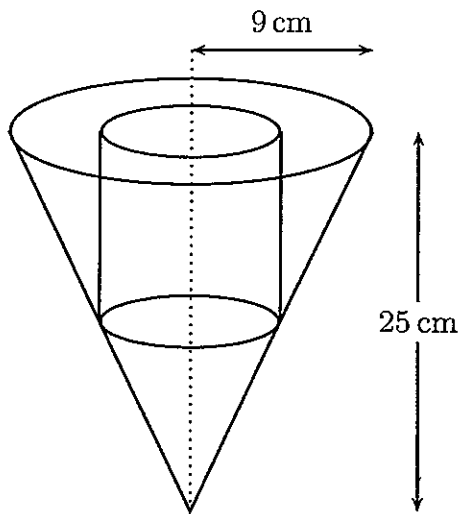
(a) Nick has found his dream home and needs to borrow \$700 000 from the bank to be able to purchase it. He has calculated that he is able to afford monthly repayments of \$6000 per month. The loan plus interest is to be repaid in equal monthly instalments of \$ M over 30 years. Reducible interest is charged at 9.6% per annum and is calculated monthly. Let \$ A_n be the amount owing after n months.

(i) Write down expressions for A_1 and A_2 , and show that the amount owing after 3 months is given by $A_3 = 700\,000(1.008)^3 - M(1 + 1.008 + 1.008^2)$. 3

(ii) Hence show that $A_n = 700\,000(1.008)^n - 125M(1.008^n - 1)$. 1

(iii) Calculate the monthly instalment \$ M , correct to the nearest dollar, and determine whether Nick will be able to purchase his dream home. 2

(b)



A cylinder is inscribed in a cone of radius 9 cm and height 25 cm.

(i) Show that the height h of the cylinder is given by 2

$$h = \frac{25(9 - r)}{9},$$

where r is the radius of the cylinder.

(ii) Find the volume V of the cylinder in terms of r . 1

(iii) Hence find the maximum possible volume of the cylinder. 3

END OF EXAMINATION

MATHEMATICS SOLUTIONS 2010

Question 1

a) $9 \cdot 36034 \dots$
 $= 9 \cdot 4$ ✓

b) $x^3 - 125$
 $= (x-5)(x^2+5x+25)$ ✓

c) LHS = $(\sqrt{7}-3)(2\sqrt{7}+2)$
 $= 14 + 2\sqrt{7} - 6\sqrt{7} - 6$
 $= 8 - 4\sqrt{7}$ ✓

$\therefore p=8$ and $q=-4$ ✓

(d) $\frac{x}{3} - \frac{x+2}{4}$
 $= \frac{4x-3(x+2)}{12}$ ✓
 $= \frac{x-6}{12}$ ✓

(e) $|x-1| < 3$
 $-3 < x-1 < 3$ ✓
 $\therefore -2 < x < 4$ ✓

(f) $\cos \theta = \frac{\sqrt{3}}{2}$
 $\theta = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ ✓✓ (1 mark if not in radians)

g) $S_{17} = \frac{17}{2} [6 + 16(8)]$ ✓
 $= 1139$ ✓

QUESTION 2

a) (i) $AB = \sqrt{(-1-3)^2 + (4-0)^2}$
 $= \sqrt{16+16}$
 $= 4\sqrt{2}$ ✓

(ii) $m = \frac{4-0}{-1-3}$
 $= -1$ ✓

iii) $y-6 = -(x-1)$
 $= -x+1$
 $x+y-7=0$ ✓
 (or $y = -x+7$)

cuts x-axis when $y=0$
 so D has coordinates (7,0) ✓

(d) $p = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$
 $= \frac{|3+0-7|}{\sqrt{1^2+1^2}}$ ✓
 $= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
 $= 2\sqrt{2}$ ✓

(v) $CD = \sqrt{(1-7)^2 + (6-0)^2}$
 $= \sqrt{36+36}$
 $= 6\sqrt{2}$ ✓

Area = $\frac{1}{2} \times 2\sqrt{2} (6\sqrt{2} + 4\sqrt{2})$
 $= \sqrt{2} \times 10\sqrt{2}$
 $= 20$ units² ✓
 (Accept other valid methods)

(b) (i) $\cos \theta = \frac{4^2+5^2-6^2}{2 \times 4 \times 5}$ ✓

$= \frac{16+25-36}{40}$
 $= \frac{5}{40}$
 $= \frac{1}{8}$ ✓

(ii) $BD^2 = 1^2+3^2 - 2 \times 4 \times 3 \times \frac{1}{8}$
 $= 16+9-3$
 $= 22$ ✓

$BD = \sqrt{22}$ ✓

QUESTION 3

(a) (i) $\frac{dy}{dx} = 6x - \frac{1}{x^2}$ ✓

(ii) $\frac{dy}{dx} = 24(2x-5)^3$ ✓

(iii) $\frac{dy}{dx} = \tan x + x \sec^2 x$ ✓

b) $y = \ln x$
 $\frac{dy}{dx} = \frac{1}{x}$

at $x=e$, $y = \frac{1}{e}$ ✓

$y-1 = \frac{1}{e}(x-e)$
 $y = \frac{1}{e}x$ ✓

(c) $\int \sec^2 \frac{1}{3} x dx$
 $= \frac{\tan \frac{1}{3} x}{\frac{1}{3}} + c$ ✓
 $= 3 \tan \frac{1}{3} x + c$ ✓

(d) $\int_0^1 \frac{4}{4x+1} dx$
 $= [\ln(4x+1)]_0^1$ ✓
 $= \ln 5 - \ln 1$
 $= \ln 5$ ✓

QUESTION FOUR

a) (i) (0,2) ✓
 (ii) (0,3) ✓

b) (i) $\angle WZY = 90^\circ - \theta$ (angle sum of triangle WZY) ✓
 $\therefore \angle WZX = \theta$ (angle XZY = 90°) ✓

(ii) $\angle WYZ = \angle WZX$ (from part (i)) ✓
 $\angle YWZ = \angle ZWX$ (right angles) ✓

$\therefore \triangle WZX \cong \triangle WYZ$ (equiangular) ✓

(iii) $\frac{XV}{WZ} = \frac{WZ}{WY}$ (matching sides of similar triangles)

$\frac{a}{5} = \frac{5}{4}$
 $\therefore a = \frac{5}{4}$ ✓

c) (i) $AP = -\frac{2}{3}$ ✓

(ii) $\frac{5(A+B)}{AP} = \frac{5(\frac{1}{3})}{-\frac{2}{3}}$
 $= \frac{5 \times 4^{-3}}{3 \times 2}$
 $= -10$ ✓

d) (i) $ar = 270$ and $ar^4 = 80$
 by solving simultaneously

$r^3 = \frac{8}{27}$
 $\therefore r = \frac{2}{3}$ ✓

$a(2/3)^4 = 270$ ✓
 $\therefore a = 405$ ✓

(ii) $S_{\infty} = \frac{405}{1/3}$
 $= 1215$ ✓

QUESTION FIVE

a) $\frac{dy}{dx} = 3x^2 - 3$

stationary at $\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 6x$

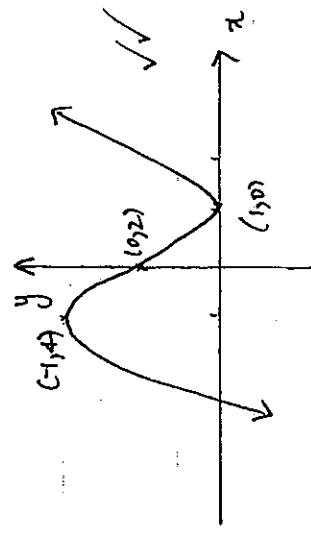
x	-1	0	1
$\frac{d^2y}{dx^2}$	-6	0	6
Concavity	∨	-	∨

when $x = -1, y = -1 + 3 + 2 = 4$ $\therefore (-1, 4)$ is a **MAXIMUM** turning point. ✓
 (* for both y-coordinates)

when $x = 1, y = 1 - 3 + 2 = 0$ $\therefore (1, 0)$ is a **MINIMUM** turning point. ✓

Point of inflexion at $\frac{d^2y}{dx^2} = 0$, and a concavity change.

$6x = 0$, when $x = 0, y = 1 - 3 + 2 = 0$ ✓
 Concavity changes (see above) ✓
 $\therefore (0, 2)$ is a point of inflexion ✓



(iv) Concave down: $x < 0$ ✓

(b) (i) $x^2 + x - 12 = -x^2 + 3x$

$2x^2 - 2x - 12 = 0$

$x^2 + x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2$ or 3

$\therefore P$ has x-coordinate -2 . ✓

(ii) Area = $\int_{-2}^3 (-x^2 + 3x - (x^2 + x - 12)) dx$

= $\int_{-2}^3 (-x^2 + 3x - x^2 - x + 12) dx$

= $\int_{-2}^3 (-2x^2 + 2x + 12) dx$ ✓

= $2 \int_{-2}^3 (-x^2 + x + 6) dx$

= $2 \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x \right]_{-2}^3$ ✓

= $2 \left[\left(-1 + \frac{9}{2} + 18 \right) - \left(\frac{8}{3} + 2 - 12 \right) \right]$

= $2 \left[\frac{21}{2} + \frac{22}{3} \right]$

= $2 \left(\frac{81 + 44}{6} \right)$

= $\frac{125}{3}$ units²

(or $41\frac{2}{3}$ units²) ✓

QUESTION SIX

i) $l = vr$
 $40 = 50\theta$ ✓
 $\theta = \frac{4}{5}$

$\therefore \angle AOD = \frac{4}{5}$ radians

ii) Area ABCD = $\frac{1}{2}OB \cdot \theta - \frac{1}{2}OA^2 \theta$
 $= \frac{1}{2} \times \frac{4}{5} (60^2 - 50^2)$ ✓
 $= \frac{2}{5} \times 1100$
 $= 440 \text{ m}^2$ ✓

b) i) $P = Ac \cdot kt$
 $\frac{dP}{dt} = k \times Ac$
 $= kP$ ✓

iii) $t=0, P=50$
 $50 = Ac$
 $\therefore A=50$
 $t=10, P=275$ ✓

$275 = 50e^{10k}$
 $e^{10k} = \frac{11}{5}$ ✓

$\ln\left(\frac{11}{5}\right) = 10k$
 $k = \frac{1}{10} \ln\left(\frac{11}{5}\right)$ ✓

iii) 3 weeks = 21 days, $t=21$
 $P = 50e^{21k}$
 $= 1793.6 \dots$ ✓

\therefore Julian will not discover an infestation. ✓

QUESTION 7

a) $\int_{1/2}^m \frac{1}{x^2} dx = 1$

$\left[-\frac{1}{x}\right]_{1/2}^m = 1$ ✓

$-1 + \frac{1}{1/2} = 1$

$-\frac{1}{m} + 2 = 1$

$m = 1$ ✓

b) $\angle BXA = \angle XAD = \alpha$ (Alternate angles, AD || BC) ✓

$\angle CXD = \angle ADX = \beta$ (Alternate angles, AD || BC) ✓

$\therefore \angle AXD = 180 - (\alpha + \beta)$ (straight angle BXC)

AX bisects $\angle BAD$

so $\angle BAX = \alpha$ ($\angle BAD = 2\alpha$)

$\therefore \angle BAX = \angle BXA$

$\therefore \triangle BAX$ is isosceles (equal base angles) ✓

ii) DX bisects $\angle CDA$, so $\angle ADC = 2\beta$

$\angle BAD + \angle ADC = 180$ (co-interior angles, AB || CD) ✓

$2\alpha + 2\beta = 180$

$\alpha + \beta = 90$

$\therefore \angle AXD = 180 - 90$

$= 90^\circ$ ✓

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	0.5554	1.5708	1.6661	0

$$\int_0^{\pi} x \sin x = \frac{\pi}{6} (0 + 4 \times 0.5554 + 1.5708) + \frac{\pi^2}{6} (1.5708 + 4 \times 1.6661 + 0)$$

$$= \frac{\pi}{12} (3.7924 + 8.2352)$$

$$= 3.1488 \dots$$

$$= 3.15$$

$$\frac{d}{dx} (\sin x - x \cos x) = \cos x - (\cos x - x \sin x)$$

$$= x \sin x$$

$$\int_0^{\pi} x \sin x dx = \left[\sin x - x \cos x \right]_0^{\pi}$$

$$= \sin \pi - \pi \cos \pi$$

$$= \pi$$

(ii) Error = approximation - exact value

$$= 3.15 - \pi$$

$$= 0.008407 \dots$$

% Error = $\frac{\text{Error}}{\pi} \times 100$

$$= 0.267 \dots$$

$$= 0.3\%$$

QUESTION EIGHT

(a) $m^2 + 7x - m = 0$

$$\alpha + \beta = -\frac{b}{a} \quad \alpha\beta = \frac{c}{a}$$

$$x = 3/4 > \frac{3}{4} \times \beta = -\frac{m}{m} \quad \beta = -\frac{4}{3}$$

b) (i) $\frac{dv}{dt} = \frac{6t}{t^2+1}$

$$V = 3 \int \frac{6t}{t^2+1} dt$$

$$= 3 \ln(t^2+1) + C$$

$t=0 \quad V=0 \quad C=0$

$$\therefore V = 3 \ln(t^2+1)$$

(ii) $V = \frac{4}{3} \pi r^3, \quad r=1.5$

$$= \frac{4}{3} \pi \times \left(\frac{3}{2}\right)^3$$

$$= \frac{9\pi}{2}$$

$$\frac{9\pi}{2} = 3 \ln(t^2+1)$$

$$\ln(t^2+1) = \frac{3\pi}{2}$$

$$t^2+1 = e^{\frac{3\pi}{2}}$$

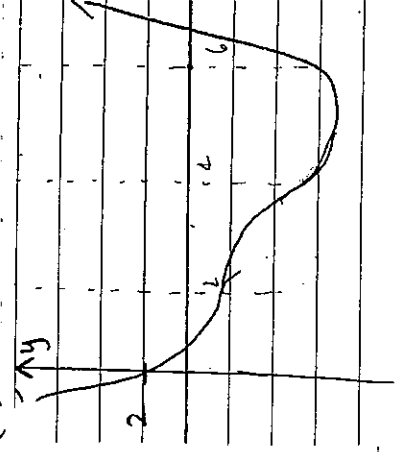
$$t^2 = e^{\frac{3\pi}{2}} - 1$$

$$t = \sqrt{e^{\frac{3\pi}{2}} - 1} \approx 10.55$$

(i) $x > 6$

(ii) $2 < x < 4$

(iii)



QUESTION NINE

a) $2\sin^2 \alpha - \cos \alpha + 1 = 0$
 $2(1 - \cos^2 \alpha) - \cos \alpha + 1 = 0$ ✓

$2 - 2\cos^2 \alpha - \cos \alpha + 1 = 0$

$2\cos^2 \alpha + \cos \alpha - 3 = 0$

$(2\cos \alpha + 3)(\cos \alpha - 1) = 0$ ✓

$\therefore \cos \alpha = -\frac{3}{2}$ or 1

$\therefore \alpha = 0, \pi$ ✓

b) $\log_6(x+3) + \log_6(x-2) = 2$

$\log_6[(x+3)(x-2)] = 2$

$(x+3)(x-2) = 36$ ✓

$x^2 + x - 6 = 36$

$x^2 + x - 42 = 0$

$(x-6)(x+7) = 0$ ✓

$x = 6$ or -7

But $\log_6(x+3)$ & $\log_6(x-2)$ must be positive ✓

$\therefore x=6$ is the only solution ✓

QUESTION 10

a) (i) $r = 9.6 \div 12 = 0.8\%$ per month ✓

$A_1 = 700,000(1.008) - M$ ✓

$A_2 = A_1(1.008) - M$
 $= (700,000(1.008) - M)(1.008) - M$

$= 700,000(1.008)^2 - M(1+1.008)$ ✓

$A_3 = A_2(1.008) - M$

$= [700,000(1.008)^2 - M(1+1.008)](1.008) - M$

$= 700,000(1.008)^3 - M(1+1.008+1.008^2)$ ✓

(ii) $A_n = 700,000(1.008)^n - M(1+1.008+\dots+1.008^{n-1})$

$= 700,000(1.008)^n - M\left(\frac{1.008^n - 1}{0.008}\right)$

$= 700,000(1.008)^n - 125M(1.008^n - 1)$ ✓

(iii) If repaid in 30 years, $A_{360} = 0$.

$700,000(1.008)^{360} - 125M(1.008^{360} - 1) = 0$

$M = \frac{700,000(1.008)^{360}}{125(1.008^{360} - 1)}$ ✓

$= 5937.119\dots$

$= \$5937$ (to the nearest dollar) ✓

\therefore Nick will be able to afford his dream home.

(c) $x = 5e^{-t} \sin t$

(i) $\frac{dx}{dt} = -5e^{-t} \sin t + 5e^{-t} \cos t$ ✓

$= 5e^{-t}(\cos t - \sin t)$

(i) $t=0, x=0, v=5$ ✓

(ii) $\frac{dx}{dt} = 0$ when $\cos t - \sin t = 0$

$\frac{\sin t}{\cos t} = 1$ ✓

$\tan t = 1$ ✓

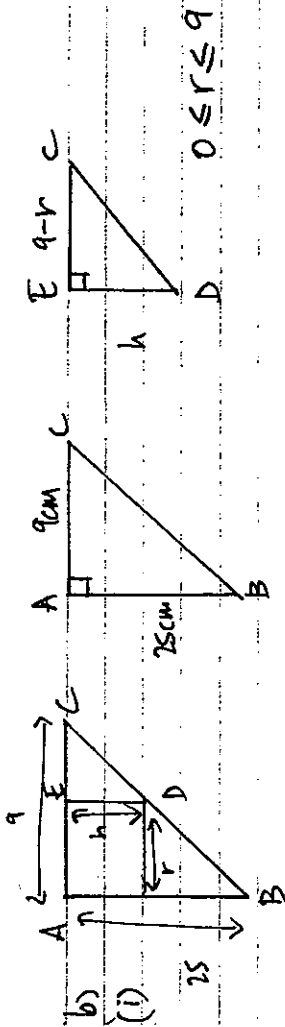
$\therefore t = \frac{\pi}{4}$

(iv) $\ddot{x} = -10e^{-t} \cos t$

acceleration is zero when

$\cos t = 0$

i.e. $t = \frac{\pi}{2}$ ✓



$\triangle ABC \sim \triangle AED$ (equiangular)

$$\frac{9}{25} = \frac{9-r}{h} \quad (\text{matching sides in similar triangles})$$

$$h = \frac{25(9-r)}{9} \quad \checkmark$$

$$\begin{aligned} \text{(ii) } V &= \pi r^2 h \\ &= \pi r^2 \times \frac{25(9-r)}{9} \\ &= \frac{25\pi}{9} (9r^2 - r^3) \quad \checkmark \end{aligned}$$

$$\text{ii) } \frac{dV}{dr} = \frac{25\pi}{9} (18r - 3r^2) \quad \text{and} \quad \frac{d^2V}{dr^2} = \frac{25\pi}{9} (18 - 6r)$$

stationary point at $\frac{dV}{dr} = 0$, $18r - 3r^2 = 0$, $3r(r-6) = 0$ \checkmark
 $r = 0$ or 6

at $r = 6$, $\frac{d^2V}{dr^2} = -50\pi < 0 \therefore$ a MAXIMUM turning point \checkmark
 (at $r = 0$, $\frac{d^2V}{dr^2} = 50\pi > 0$, so a MIN)

$$\text{Max Volume} = \frac{25\pi \times 36 \times 3}{9}$$

$$= 300\pi \text{ cm}^3 \quad \checkmark$$